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Available in alternate formats upon request.
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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the Answer/Scoring Sheet are identical
- students and markers only use a pencil to complete the Answer/Scoring Sheets
- the totals of each of the four parts are written at the bottom
- each student’s final result is recorded, by booklet number, on the corresponding Answer/Scoring Sheet
- the Answer/Scoring Sheet is complete
- a photocopy has been made for school records

Once marking is completed, please forward the Answer/Scoring Sheets to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section “Booklet 2 Questions.”

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student’s response must be complete and correct. Where alternative answering methods are possible, the Marking Guide attempts to address the most common solutions. For general guidelines regarding the scoring of students’ responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an Answer/Scoring Sheet is marked with “0” and/or “NR” only (e.g., student was present but did not attempt any questions) please document this on the Irregular Test Booklet Report.
**Assistance**

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

Allison Potter  
Assessment Consultant  
Grade 12 Pre-Calculus Mathematics  
Telephone: 204-945-7590  
Toll-Free: 1-800-282-8069, extension 7590  
Email: allison.potter@gov.mb.ca
Information for Markers

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the Answer/Scoring Sheet that represents the marks awarded based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the Answer/Scoring Sheet in a separate section. There is a $\frac{1}{2}$ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e. committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example:

A student has a preliminary mark of 72. The student committed two E1 errors ($\frac{1}{2}$ mark deduction), four E7 errors ($\frac{1}{2}$ mark deduction), and one E8 error ($\frac{1}{2}$ mark deduction). Although seven communication errors were committed in total, there is a deduction of only $1\frac{1}{2}$ marks.

<table>
<thead>
<tr>
<th>COMMUNICATION ERRORS / ERREURS DE COMMUNICATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).</td>
</tr>
<tr>
<td>Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).</td>
</tr>
<tr>
<td>E1 ● E2 ○ E3 ○ E4 ○ E5 ○ E6 ○ E7 ● E8 ● E9 ○ E10 ○</td>
</tr>
</tbody>
</table>

Mark assigned to the student / Note accordée à l’élève

<table>
<thead>
<tr>
<th>Booklet 1 / Cahier 1 + Multiple Choice / Choix multiple</th>
<th>Booklet 2 / Cahier 2 - Communication Errors / Erreurs de communication</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 +</td>
<td>40 - $\frac{1}{2}$</td>
<td>70 1/2</td>
</tr>
<tr>
<td>31</td>
<td>49</td>
<td>88</td>
</tr>
</tbody>
</table>

Maximum deduction of 5 marks / déduction maximale de 5 points
Scoring Guidelines
Question 1

Gina correctly started to answer the following question. Complete her solution.

**Question:** Solve the following equation for all real values of $\theta$.
Express your answer in radians correct to 3 decimal places.

$$3 \sin^2 \theta - 14 \sin \theta - 5 = 0$$

**Gina’s solution:**

$$3 \sin^2 \theta - 14 \sin \theta - 5 = 0$$

$$(3 \sin \theta + 1)(\sin \theta - 5) = 0$$

**Solution**

**Method 1**

$$3 \sin^2 \theta - 14 \sin \theta - 5 = 0$$

$$(3 \sin \theta + 1)(\sin \theta - 5) = 0$$

$$\sin \theta = -\frac{1}{3} \quad \sin \theta = 5$$

$$\theta_r = 0.339\,837$$

$$\theta = 3.481\,429,\,5.943\,348$$

$$\frac{1}{2} \text{ mark for } \sin \theta = 5$$

$$\frac{1}{2} \text{ mark for no solution}$$

$$\frac{1}{2} \text{ mark for each value of } \theta$$

$$\theta = 3.481 + 2k\pi, \, k \in \mathbb{I}$$

$$\theta = 5.943 + 2k\pi, \, k \in \mathbb{I}$$

1 mark for general solution

3 marks

---

**Note(s):**

- give maximum of 2 marks for solution in degrees:

$$\theta = 199.471^\circ + 360^\circ k, \, k \in \mathbb{I}$$

$$\theta = 340.529^\circ + 360^\circ k, \, k \in \mathbb{I}$$
**Question 1**

**Method 2**

\[ y = 3 \sin^2 \theta - 14 \sin \theta - 5 \]  

½ mark for equation

or

Find all zeros over the reals.

\[ \theta = 3.481 + 2k\pi, \ k \in \mathbb{Z} \]  

1 mark for solutions

\[ \theta = 5.943 + 2k\pi, \ k \in \mathbb{Z} \]  

1 mark for general solution

3 marks
Question 2

Find and simplify the 6th term in the binomial expansion of \( (3x^4 - \frac{1}{x^3})^9 \).

**Solution**

\[
t_6 = \binom{9}{5} (3x^4)^4 \left( -\frac{1}{x^3} \right)^5
\]

\[
= (126) \left( 81x^{16} \right) \left( -\frac{1}{x^{15}} \right)
\]

\[
= -10206x
\]

2 marks (1 mark for \( \binom{9}{5} \), ½ mark for each consistent factor)

1 mark for simplification (½ mark for evaluating coefficient, ½ mark for simplifying variable)

3 marks
The number of times a website is visited can be modeled by the function:

\[ A = 800(e)^{rt} \]

where \( A \) = the total number of visitors at time \( t \)
\( t \) = the time in days \((t \geq 0)\)
\( r \) = the rate of growth

After 5 days, 40 000 people have visited the site.
Determine the number of visitors expected after 9 days.
Express your answer as a whole number.

**Solution**

**Method 1**

\[
\frac{40 000}{800} = e^{5r}
\]

\[
\frac{40 000}{800} = e^{5r}
\]

\[
\ln 50 = \ln (e^{5r})
\]

\[
\ln 50 = 5r
\]

\[
\frac{\ln 50}{5} = r
\]

\[
r = 0.782 404 601
\]

\[
A = 800(e^{(0.782404601)(9)})
\]

Express your answer as a whole number.

**3 marks**
Question 3

Method 2

Use calculator to find the value of $r$.

\[ y = 40\,000 \]
\[ y = 800e^{5x} \]

\[ \frac{1}{2} \text{ mark for equations} \]

Find the point of intersection of these two functions.

\[ x = 0.782\,404\,601 \]

\[ \frac{1}{2} \text{ mark for justification} \]

1 mark for finding the value of $x$ at the intersection point

Substitute $x$ for $r$ into formula.

\[ A = 800e^{(0.782\,404\,601)(9)} \]

\[ A = 914\,610.103 \]

\[ A = 914\,610 \]

1 mark for value of $A$

3 marks
Question 4

Solve algebraically:

\[10^{3x} = 7^{x+5}\]

Express your answer correct to 3 decimal places.

**Solution**

**Method 1**

\[10^{3x} = 7^{x+5}\]

\[\log 10^{3x} = \log 7^{x+5}\]

\[3x \log 10 = (x + 5) \log 7\]

\[3x \log 10 = x \log 7 + 5 \log 7\]

\[3x \log 10 - x \log 7 = 5 \log 7\]

\[x = \frac{5 \log 7}{3 \log 10 - \log 7}\]

\[x = 1.960\ 873\]

\[x = 1.961\]

\[\frac{1}{2} \text{ mark for applying logs}\]

\[1 \text{ mark for power rule}\]

\[\frac{1}{2} \text{ mark for collecting terms with } x\]

\[\frac{1}{2} \text{ mark for solving for } x\]

\[\frac{1}{2} \text{ mark for evaluating quotient of logarithms}\]

**Method 2**

\[10^{3x} = 7^{x+5}\]

\[\log 10^{3x} = \log 7^{x+5}\]

\[3x \log 10 = (x + 5) \log 7\]

\[3x \log 10 = x \log 7 + 5 \log 7\]

\[3x - x \log 7 = 5 \log 7\]

\[x = \frac{5 \log 7}{3 - \log 7}\]

\[x = 1.960\ 873\]

\[x = 1.961\]

3 marks
A word contains two Ms, two Es, two Ns, and no other repeated letters. Suppose one of the Ns is replaced by an M. Will this replacement result in greater or fewer permutations? Justify your reasoning.

**Solution**

**Method 1**

\[
\begin{align*}
2 \text{ Ms, 2 Es, 2 Ns:} & \quad \frac{(\text{total number of letters})!}{2!2!2!} \\
& \quad \frac{(\text{total number of letters})!}{8} \\
3 \text{ Ms, 2 Es, 1 N:} & \quad \frac{(\text{total number of letters})!}{3!2!1!} \\
& \quad \frac{(\text{total number of letters})!}{12}
\end{align*}
\]

If one of the Ns is changed to an M, there would be fewer permutations.

**Method 2**

If one of the Ns is changed to an M, there would be fewer permutations because you would be dividing the total number of letters by a larger number.

2 marks for justification
Question 6

There is a group of 16 boys and 12 girls. How many ways can a committee of 3 people be formed if there must be at least 2 girls on the committee?
Express your answer as a whole number.

Solution

Method 1

Case 1: 2 girls, 1 boy
\[ \binom{12}{2} \cdot \binom{16}{1} = 1056 \]
1 mark for case 1 (½ mark for each factor shown in a product)

Case 2: 3 girls
\[ \binom{12}{3} = 220 \]
1 mark for case 2

\[ 1056 + 220 = 1276 \]
1 mark for addition of cases

3 marks

Method 2

Case 1: 2 boys, 1 girl
\[ \binom{16}{2} \cdot \binom{12}{1} = 1440 \]
1 mark for case 1 (½ mark for each factor shown in a product)

Case 2: 3 boys
\[ \binom{16}{3} = 560 \]
1 mark for case 2

All possibilities
\[ \binom{28}{3} = 3276 \]
½ mark for all possible cases

\[ 3276 - 1440 - 560 = 1276 \]
½ mark for subtraction from total

3 marks
A student is using the formula $s = \theta r$ to find an arc length of a circle. Given a central angle measure of $35^\circ$ and a radius of 6 cm, the student’s solution is as follows:

$$s = (35)(6)$$
$$s = 210 \text{ cm}$$

Explain why this solution is incorrect.

Write the correct solution.

**Solution**

When using the formula $s = \theta r$, the angle must be in radians.  

$$35^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{35\pi}{180} \text{ or } \frac{7\pi}{36}$$

$$\therefore s = \theta r$$

$$= \frac{7\pi}{36}(6)$$

$$= \frac{42\pi}{36} \text{ cm or } \frac{7\pi}{6} \text{ cm or } 3.665 \text{ cm}$$

Note(s):

- award 1 mark for correct final answer with no work
- ½ mark for explanation
- 1 mark for conversion to radians
- ½ mark for simplification

2 marks
Given the graph of \( f(x) \) below, explain how you would sketch the graph of \( y = |f(x)| \).

**Solution**

The negative \( y \) values are reflected over the \( x \)-axis.  

1 mark for explanation
Question 9

Claire correctly solves the following equation:

\[ \log_2 (6 - x) + \log_2 (3 - x) = 2 \]

She finds two possible values of \( x \): \( x = 2 \) and \( x = 7 \).
Identify which one of these values is unacceptable and explain why.

Solution

If \( x \) is greater than 3, you have a negative argument \( \therefore x = 2 \) but \( x \neq 7 \).

or

The domain is restricted to values of \( x < 3 \): \( \therefore x = 2 \).
Question 10

Given the graph of the function \( f(x) \) below, state the domain of \( y = \sqrt{f(x)} \).

Solution

The domain is \( \{x \in \mathbb{R} \mid x \leq -1 \text{ or } x \geq 3\} \).

or

The domain is \( (-\infty, -1] \cup [3, \infty) \).

1 mark for domain

1 mark
Question 11

A school offers 4 different Science courses, 3 different Mathematics courses, and 2 different English courses.

Julie must select 1 Science course, 1 Mathematics course, and 1 English course. She thinks this creates 9 options for her timetable.

Show why Julie is incorrect.

Solution

Julie should have multiplied the number of options.

or

\[4 \times 3 \times 2 = 24 \text{ different options}\]
Question 12

Explain how Pascal’s triangle can be used to determine the coefficients in the binomial expansion of \((x + y)^n\).

**Solution**

The \((n + 1)^{st}\) row of Pascal’s triangle corresponds to the coefficients of the terms in the expansion of the binomial \((x + y)^n\).
Question 13

Prove the identity below for all permissible values of $x$:

\[
\frac{1 + \cos 2x}{\sin 2x} = \cot x
\]

Solution

Method 1

LHS $= \frac{1 + 2 \cos^2 x - 1}{2 \sin x \cos x}$

$= \frac{2 \cos^2 x}{2 \sin x \cos x}$

$= \frac{\cos x}{\sin x}$

$= \cot x$

$= \text{RHS}$

3 marks

Method 2

LHS $= \frac{1 + 1 - 2 \sin^2 x}{2 \sin x \cos x}$

$= \frac{2 - 2 \sin^2 x}{2 \sin x \cos x}$

$= \frac{1 - \sin^2 x}{\sin x \cos x}$

$= \frac{\cos^2 x}{\sin x \cos x}$

$= \frac{\cos x}{\sin x}$

$= \cot x$

$= \text{RHS}$

3 marks
Solution

Method 3

\[
\text{LHS} = \frac{1 + \cos^2 x - \sin^2 x}{2 \sin x \cos x}
\]

\[
= \frac{2 \cos^2 x}{2 \sin x \cos x}
\]

\[
= \frac{\cos x}{\sin x}
\]

\[
= \cot x
\]

\[
= \text{RHS}
\]

\[
\frac{1}{2} \text{ mark for identity}
\]

\[
\frac{1}{2} \text{ mark for identity}
\]

\[
\frac{1}{2} \text{ mark for identity (} 1 - \sin^2 x = \cos^2 x \text{)}
\]

\[
1 \text{ mark for simplification}
\]

\[
\frac{1}{2} \text{ mark for identity}
\]

3 marks
Your classmate, Leo, was absent for one of his math lessons.

Explain to Leo how to determine the cosecant ratio for an angle in standard position given that \( P(-3, -4) \) is a point on the terminal arm of the angle.

**Solution**

**Method 1**

- Locate the point \((-3, -4)\) on the coordinate plane. 
- Draw a right-angle triangle by connecting the point \((-3, -4)\) to the \(x\)-axis and to the origin. 
- The angle created with the \(x\)-axis and the line connecting the point with the origin (the terminal arm) is \(\theta\). 
- Determine the length of the terminal arm by using the Pythagorean Theorem. ½ mark
- Once you have the lengths of all three sides of the triangle, find \(\sin \theta\). ½ mark
- Invert the \(\sin \theta\) ratio to find \(\csc \theta\). ½ mark

2 marks

**Method 2**

- Determine the value of \(r\) using the Pythagorean Theorem. ½ mark
- Determine the value of \(\sin \theta \left( \sin \theta = \frac{y}{r} \right)\). ½ mark
- Identify the correct quadrant. ½ mark
- Invert the \(\sin \theta\) ratio to find \(\csc \theta\). ½ mark

2 marks

**Note(s):**

- award a maximum of 1 mark for correct work shown without an explanation in words
Question 15

Given the following graphs:

Sketch the graph of \( f(x) + g(x) \).

**Solution**

2 marks (½ mark for each point where the graph changes direction)
\[ [(-2, -2); (0, 4); (1, 4); (4, 1)] \]

**Note(s):**
- deduct a maximum of 1 mark for incorrect domain
## Answer Key for Multiple-Choice Questions

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Learning Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>D</td>
<td>R3</td>
</tr>
<tr>
<td>17</td>
<td>D</td>
<td>T1</td>
</tr>
<tr>
<td>18</td>
<td>A</td>
<td>R7</td>
</tr>
<tr>
<td>19</td>
<td>C</td>
<td>T6</td>
</tr>
<tr>
<td>20</td>
<td>C</td>
<td>R14</td>
</tr>
<tr>
<td>21</td>
<td>D</td>
<td>P4</td>
</tr>
<tr>
<td>22</td>
<td>A</td>
<td>R6</td>
</tr>
<tr>
<td>23</td>
<td>B</td>
<td>P2</td>
</tr>
<tr>
<td>24</td>
<td>D</td>
<td>R9</td>
</tr>
</tbody>
</table>
Question 16

If \((2, 3)\) is a point on the graph of \(y = f(x)\), what point must be on the graph of \(y = 3f\left(\frac{1}{4}x\right)\)?

a) \(\left(\frac{1}{2}, 1\right)\)  
   b) \(\left(\frac{1}{2}, 9\right)\)  
   c) \((8, 1)\)  
   d) \((8, 9)\)

Question 17

Consider the arc drawn on each circle. Which arc measure is closest to 3 radians?

a)  
   b)  
   c)  
   d)  

Question 18

If \(\log_2 x = 4\), then \(\log_2 (2x)\) is equal to:

a) 5  
   b) 8  
   c) 16  
   d) 32
Question 19

Simplify the following expression:
\[ \cos^2 x \left( 1 + \cot^2 x \right) \]

a) \( \sin^2 x \)
b) \( \cos^2 x \)
c) \( \cot^2 x \)
d) \( \sec^2 x \)

Question 20

Identify the graph of the function \( y = \frac{x}{x} \).

a) 

b) 

c) 

d)
Question 21

How many terms are in the expansion of $\left(3y^2 - 4z\right)^7$?

a) 2  b) 6  c) 7  d) 8

Question 22

Determine one possible restriction for the domain of $y = (x + 3)^2 - 4$ so that its inverse is a function.

a) $x \leq -3$  b) $x \leq 0$  c) $x \leq 3$  d) $x \leq 4$

Question 23

Find the total possible number of arrangements for 7 adults and 3 children seated in a row if the 3 children must sit together.

a) $10!$  b) $8!3!$  c) $7!3!$  d) $7!$

Question 24

Identify the value of the $x$-intercept of the function $y = \ln(x - 2)$.

a) $-1$  b) 0  c) 2  d) 3
Question 25

Given \( \log_b a = 3 \), give one example of possible values for \( a \) and \( b \) that make this equation true.

**Solution**

Answers will vary but \( b^3 = a \).

Some possible solutions are: 

<table>
<thead>
<tr>
<th>Solution 1</th>
<th>Solution 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 8 )</td>
<td>( a = 64 )</td>
</tr>
<tr>
<td>( b = 2 )</td>
<td>( b = 4 )</td>
</tr>
</tbody>
</table>

or

<table>
<thead>
<tr>
<th>Solution 3</th>
<th>Solution 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 27 )</td>
<td>( a = 64 )</td>
</tr>
<tr>
<td>( b = 3 )</td>
<td>( b = 4 )</td>
</tr>
</tbody>
</table>

1 mark

Question 26

The range of the graph of \( y = f(x) \) is \([-3, 2]\).

Explain why there is no effect on the range of the graph that is a result of the transformation \( y = f(-x) \).

**Solution**

\( y = f(-x) \) is a reflection over the \( y \)-axis.

The domain is affected, but the range remains the same.  

1 mark for explanation
Question 27

Sketch the graph of \( y = (x + 1)(x - 2)^2(x + 5) \).
Identify the \( x \)-intercepts and \( y \)-intercept.

**Solution**

\( x \)-intercepts: \(-5\), \(-1\), and \(2\)

\( y \)-intercept: \(20\)

**Note(s):**
- if no graph is shown, give \( \frac{1}{2} \) mark for \( x \)-intercepts \((-5\), \(-1\), and \(2\)) and \( \frac{1}{2} \) mark for \( y \)-intercept \(20\)
- relative maximum and minimum are not required

1 mark for \( x \)-intercepts
1 mark for \( y \)-intercept
1 mark for multiplicity of a zero at \( x = 2 \)

3 marks
Question 28

The graph of the function \( y = \sin x \) has been transformed to create a new graph.

The range of this new graph is \([-4, 4]\) and the zeros are \( x = k \frac{\pi}{2} \), where \( k \) is an integer.

Write the equation that corresponds to this new graph.

**Solution**

\[
\begin{align*}
\text{Amplitude} &= 4 & \text{1 mark for correct amplitude} \\
\text{Period} &= \pi & \text{½ mark for correct period} \\
\therefore b &= \frac{2\pi}{\pi} = 2 & \text{½ mark for consistent value of } b \\
y &= 4 \sin (2x) \\
\text{or} \\
y &= -4 \sin (2x)
\end{align*}
\]

Note(s):
- deduct ½ mark for a translation that results in an incorrect range and/or zeros

Question 29

Given the functions \( f(x) = x^2 - 1 \) and \( g(x) = x + 1 \), state the domain of \( \frac{g(x)}{f(x)} \).

**Solution**

Domain: \( x \in \mathbb{R} \) where \( x \neq 1 \) and \( x \neq -1 \)  \hspace{1cm} 1 mark (½ mark for \( x \neq 1 \), ½ mark for \( x \neq -1 \))
Question 30

a) Sketch the graph of \( y = 3^x \).

b) Explain how the graph of \( y = 3^x \) can be used to sketch the graph of \( y = \log_3 x \).

Solutions

a) To graph \( y = \log_3 x \), you can reflect the graph of \( y = 3^x \) over the line \( y = x \).

\[
\begin{align*}
\frac{1}{2} \text{ mark for increasing exponential function} \\
\frac{1}{2} \text{ mark for y-intercept at (0, 1)} \\
\frac{1}{2} \text{ mark for consistent point on exponential function} \\
\frac{1}{2} \text{ mark for asymptotic behaviour} \\
\text{2 marks}
\end{align*}
\]

b) To graph \( y = \log_3 x \), you can reflect the graph of \( y = 3^x \) over the line \( y = x \).

\[
\begin{align*}
\text{or}
\end{align*}
\]

You can switch the \( x \) and \( y \) coordinates of \( y = 3^x \) to get the graph of \( y = \log_3 x \).

\[
\begin{align*}
1 \text{ mark for explanation}
\end{align*}
\]

Note(s):
- in (b), give \( \frac{1}{2} \) mark for having only stated they are inverse functions of each other
A box in the shape of a rectangular prism has side lengths $x$, $x + 2$, and $x + 10$.

Write a function, $V(x)$, to express the volume of the box in terms of $x$.

Find all possible values of $x$, given that the volume of the box is 96 cm$^3$.

State the dimensions of the box.

**Solution**

$$V(x) = (x)(x + 2)(x + 10)$$

$$96 = (x)(x + 2)(x + 10)$$

$$96 = x^3 + 12x^2 + 20x$$

$$0 = x^3 + 12x^2 + 20x - 96$$

when $x = 2$:

$$0 = (2)^3 + 12(2)^2 + 20(2) - 96$$

$$0 = 8 + 48 + 40 - 96$$

$$0 = 0$$

$\therefore$ $x = 2$ is a possible value.

The dimensions of the box are 2 cm $\times$ 4 cm $\times$ 12 cm.
Question 32

Given the graph of \( f(x) \) below, sketch the graph of \( y = -f(x) \).

Solution

Determine the coordinates of a point \((x, y)\) on the unit circle if you are given \( \theta = 30^\circ \) where \( \theta \) is in standard position.

Solution

\[ P(\theta) = (\cos \theta, \sin \theta) \]

If \( \theta = 30^\circ \), then the coordinates of \( P(\theta) \) would be \( \left( \cos 30^\circ, \sin 30^\circ \right) \).

or

\[ P(30^\circ) = \left( \frac{\sqrt{3}}{2}, \frac{1}{2} \right) \]

1 mark for answer stated as coordinate point
Question 34

Given the following sinusoidal equation:

\[ P(t) = 3000 \sin \left( \frac{\pi}{10} (t - 2010) \right) + 10000 \]

Determine the maximum value of \( P(t) \) and a value of \( t \) at which this maximum occurs.

Solution

Method 1

Maximum value \( = 10000 + 3000 \)

\( = 13000 \)  \hspace{1cm} 1 mark for maximum value

\[
13000 = 3000 \sin \left( \frac{\pi}{10} (t - 2010) \right) + 10000
\]

\[
3000 = 3000 \sin \left( \frac{\pi}{10} (t - 2010) \right)
\]

\[
1 = \sin \left( \frac{\pi}{10} (t - 2010) \right)
\]  \hspace{1cm} ½ mark for simplifying

\[
\frac{\pi}{2} = \frac{\pi}{10} (t - 2010)
\]

\( 5 = t - 2010 \)

\( t = 2015 \)  \hspace{1cm} ½ mark for solving for \( t \)

3 marks

Note(s):

- the period of the function is 20 \( \therefore \) other acceptable answers are: \( t = 2015 \pm 20 \)
Solution
Method 2

\[ P(t) = 13000 \]
\[ t = 2015 \]
\[ \therefore \text{the maximum value is 13000 when } t = 2015. \]

1 mark for period

1 mark for maximum value

1 mark for solving for \( t \)

3 marks
Question 35

Sketch the graph of \( y = \sqrt{2x - 2} \).

Solution

Method 1

\[
y = \sqrt{2x - 2} \\
= \sqrt{2(x - 1)}
\]

1 mark for domain: \([1, \infty)\)
1 mark for shape (graph of a radical function)
1 mark for horizontal compression
Method 2

1 mark for domain of \( y = \sqrt{2x - 2} : [1, \infty) \)
1 mark for invariant points where \( y = 0 \) and \( y = 1 \) (½ mark for each point)
½ mark for graph of \( y = \sqrt{2x - 2} \) drawn above the graph of \( y = 2x - 2 \) between the invariant points
½ mark for graph of \( y = \sqrt{2x - 2} \) drawn below the graph of \( y = 2x - 2 \) after the invariant point
where \( y = 1 \)

3 marks
Question 36

Given \( f(x) = 2x - 6 \), write the equation of \( f^{-1}(x) \).

**Solution**

Switch the \( x \) and \( y \) values.

\[
x = 2y - 6 \\
\Rightarrow x + 6 = 2y \\
\Rightarrow \frac{x + 6}{2} = y \\
\therefore f^{-1}(x) = \frac{x + 6}{2}
\]

1 mark for switching \( x \) and \( y \) values

\[
\frac{x + 6}{2} = y
\]

\[
\therefore f^{-1}(x) = \frac{x + 6}{2}
\]

½ mark for solving for \( y \)

½ mark for writing equation of \( f^{-1}(x) \)

2 marks

Question 37

Frank tried to expand a logarithmic expression using the laws of logarithms. He made one error.

**Frank's solution:** \[
\log_a \left( \frac{x + 2}{zw} \right) = \log_a x + \log_a 2 - \log_a z - \log_a w
\]

Write the correct solution.

**Solution**

Correct solution: \[
\log_a \left( \frac{x + 2}{zw} \right) = \log_a (x + 2) - \log_a z - \log_a w
\]

1 mark for correct solution
Determine all non-permissible values of $\theta$ over the interval $[0, 2\pi]$.

$$\frac{\sin \theta}{1 + \cos \theta} + \csc \theta + \cot \theta$$

Explain your reasoning.

**Solution**

To determine non-permissible values, the denominator needs to equal zero.

The denominators of this expression are “$1 + \cos \theta$” and “$\sin \theta$” (since $\csc \theta = \frac{1}{\sin \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$).

$$\therefore 1 + \cos \theta = 0 \quad \sin \theta = 0$$

$$\cos \theta = -1 \quad \theta = 0, \pi, 2\pi$$

$$\theta = \pi$$

$$\therefore$$ the non-permissible values of $\theta$ over $[0, 2\pi]$ are $0, \pi, \text{ and } 2\pi$. 1 mark for all non-permissible values of $\theta$ (½ mark for each equation)
Question 39

Given the following graphs:

![Graphs of f(x) and g(x)]

a) Determine the value of \( (f \cdot g)(0) \).

b) Determine the value of \( g(f(4)) \).

c) Determine a value for \( k \) where \( f(k) = 1 \).

**Solutions**

a) \( f(0) = -1 \)
\( g(0) = 2 \)

\[
(f \cdot g)(0) = (-1)(2) = -2
\]

1 mark for the value of \( (f \cdot g)(0) \)

b) \( f(4) = -1 \)
\( g(-1) = 3 \)

\[
\frac{1}{2} \text{ mark for } f(4)
\]

\[
\frac{1}{2} \text{ mark for } g(f(4)) \text{ consistent with } f(4) \text{ value}
\]

1 mark


c) \( k = 2 \) or \( k = -3 \)

1 mark for a value for \( k \)
Question 40

Given that \( h(x) = 2x^2 + 5x - 3 \) and that \( h(x) = f(x) \cdot g(x) \), determine \( f(x) \) and \( g(x) \).

**Solution**

\[
f(x) = 2x - 1 \quad \text{1 mark for two correct factors of } h(x)
\]
\[
g(x) = x + 3
\]

Other answers are possible.

Question 41

The graph of \( y = 2 \cos \theta + 1 \) below can be used to solve the equation \( \cos \theta = -\frac{1}{2} \) over the interval \([ -2\pi, 2\pi ]\). Indicate on the graph where to find the solutions to the equation \( \cos \theta = -\frac{1}{2} \).

**Solution**

The solution to the equation \( \cos \theta = -\frac{1}{2} \) is found where the graph of \( y = 2 \cos \theta + 1 \) crosses the \( x \)-axis.

\[
\theta
\]
\[
y = 2 \cos \theta + 1
\]

Note(s):
- give \( \frac{1}{2} \) mark for indicating 1, 2, or 3 of the solutions
Question 42

The function \( f(x) \) is transformed.

A new function, \( y = \frac{1}{f(x)} \), is created that does not have any vertical asymptotes.

What can you conclude about the original function \( f(x) \)?

**Solution**

If \( f(x) \) does not have any \( x \)-intercepts, then the transformation would not have any vertical asymptotes.

or

\[ f(x) \text{ cannot equal zero.} \]

or

\[ f \neq 0 \]

**Note(s):**

- award \( \frac{1}{2} \) mark for a correct example without a given conclusion

Question 43

Draw the angle \( -\frac{7\pi}{8} \) in standard position.

**Solution**

\[ \text{1 mark for angle drawn in Quadrant III} \]
Determine the exact value of:

\[ 4 \cos \left( \frac{11\pi}{12} \right) \]

**Solution**

**Method 1**

\[ 4 \cos \left( \frac{11\pi}{12} \right) = 4 \cos \left( \frac{\pi}{4} + \frac{2\pi}{3} \right) \]

\[ = 4 \left[ \cos \frac{\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{\pi}{4} \sin \frac{2\pi}{3} \right] \]

\[ = 4 \left[ \left( \frac{\sqrt{2}}{2} \right) \left( -\frac{1}{2} \right) - \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) \right] \]

\[ = 4 \left[ -\frac{\sqrt{2} - \sqrt{6}}{4} \right] \]

\[ = -\sqrt{2} - \sqrt{6} \]

1 mark for combination
2 marks (½ mark for each exact value)

3 marks
Solution

Method 2

\[ \frac{11\pi}{12} \] has a reference angle of \( \frac{\pi}{12} \).

\[ \cos \left( \frac{\pi}{12} \right) = \cos \left( \frac{\pi}{4} - \frac{\pi}{6} \right) \]

\[ = \cos \frac{\pi}{4} \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \sin \frac{\pi}{6} \]

\[ = \left( \frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) + \left( \frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) \]

\[ = \frac{\sqrt{6} + \sqrt{2}}{4} \]

Since \( \frac{11\pi}{12} \) is in Quadrant II, cosine is negative.

\[ \therefore 4 \cos \left( \frac{11\pi}{12} \right) = 4 \left( \frac{-\sqrt{6} - \sqrt{2}}{4} \right) \]

\[ = -\sqrt{6} - \sqrt{2} \]

\[ \text{3 marks} \]

Note(s):

- \( \frac{-2 - 2\sqrt{3}}{\sqrt{2}} \) is an acceptable way to write the solution
- in Method 1, another possible combination is \( \left( \frac{\pi}{6} + \frac{3\pi}{4} \right) \)
- deduct \( \frac{1}{2} \) mark if answer is not expressed as a single fraction
- in Method 2, the reference angle need not be explicitly stated in order to obtain full marks
Question 45

Sketch the graph of $f(x) = \frac{x - 4}{x^2 - 3x - 4}$.

**Solution**

$$f(x) = \frac{x - 4}{x^2 - 3x - 4} = \frac{x - 4}{(x - 4)(x + 1)} = \frac{1}{x + 1} \text{ with a point of discontinuity at } x = 4$$

Point of discontinuity: $f(4) = \frac{1}{5}$

$\therefore$ there is a point of discontinuity at $\left(4, \frac{1}{5}\right)$

$y$-intercept: $f(0) = \frac{0 - 4}{(0)^2 - 3(0) - 4}$

$= \frac{-4}{-4}$

$= 1$

1 mark for vertical asymptote at $x = -1$

½ mark for graph left of vertical asymptote

½ mark for graph right of vertical asymptote

1 mark for point of discontinuity at $x = 4$

3 marks
Question 46

Estimate the value of \( \log_5 35 \).

Justify your answer.

**Solution**

\[ 5^2 = 25 \quad \quad \quad 5^3 = 125 \]

\[ \frac{2}{2} \text{ mark for justification} \]

The value of \( \log_5 35 \) is more than 2 but less than 2.5.

\[ \frac{1}{2} \text{ mark for estimated answer} \]

1 mark

Question 47

If \( p(x) = x^5 - 12x + 1 \), determine the remainder when \( p(x) \) is divided by \( (x + 2) \).

**Solution**

**Method 1**

\[ p(x) = x^5 - 12x + 1 \]

\[ p(-2) = (-2)^5 - 12(-2) + 1 \]

\[ = -32 + 24 + 1 \]

\[ = -7 \]

1 mark for substitution

1 mark

**Method 2**

\[ -2 | 1 \ 0 \ 0 \ 0 \ -12 \ 1 \]

1 mark for correct set-up of synthetic division

\[ -2 | 1 \ 0 \ 0 \ -12 \ 1 \]

\[ \downarrow -2 \ 4 \ -8 \ 16 \ -8 \]

\[ 1 \ -2 \ 4 \ -8 \ 4 \ -7 \]

1 mark

The remainder is \(-7\).
Question 48

Describe the effects on the graph of \( y = f(x) \) when asked for the graph of \( y = f(x - 3) + 5. \)

Solution

Shift right 3 and up 5  
½ mark for horizontal translation
½ mark for vertical translation

\( (x + 3, y + 5) \)  
1 mark
Question 49

Find the exact value of the following expression:

\[ \sin \left( \frac{11\pi}{3} \right) \cdot \sec \left( \frac{4\pi}{3} \right) \cdot \tan \left( -\frac{5\pi}{6} \right) \]

**Solution**

\[
= \left( -\frac{\sqrt{3}}{2} \right) \left( -2 \right) \left( \frac{1}{\sqrt{3}} \right) \\
= 1
\]

1 mark for \( \sin \left( \frac{11\pi}{3} \right) \) (½ mark for quadrant, ½ mark for value)

1 mark for \( \sec \left( \frac{4\pi}{3} \right) \) (½ mark for quadrant, ½ mark for value)

1 mark for \( \tan \left( -\frac{5\pi}{6} \right) \) (½ mark for quadrant, ½ mark for value)

3 marks
MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the Answer/Scoring Sheet.

| E1 | answer given as a complex fraction
|    | final answer not stated
|    | answer stated in degrees instead of radians or vice versa
| E2 | changing an equation to an expression
|    | equating the two sides when proving an identity
| E3 | variable omitted in an equation or identity
|    | variables introduced without being defined
| E4 | “\(\sin^2 x\)” written instead of “\(\sin^2 x\)”
|    | missing brackets but still implied
| E5 | missing units of measure
|    | incorrect units of measure
| E6 | rounding error
|    | rounding too early
| E7 | transcription error
|    | notation error
| E8 | answer included outside the given domain
|    | bracket error made when stating domain or range
|    | domain or range written in incorrect order
| E9 | incorrect or missing endpoints or arrowheads
|    | scale values on axes not indicated
|    | coordinate points labelled incorrectly
| E10| asymptotes drawn as solid lines
    | graph crosses or curls away from asymptotes
IRREGULARITIES IN PROVINCIAL TESTS
A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all "NR") or only incorrect responses ("0")

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.
Irregular Test Booklet Report

Test: 

Date marked: 

Booklet No.: 

Problem(s) noted: 

Question(s) affected: 

Action taken or rationale for assigning marks: 

Pre-Calculus Mathematics: Marking Guide (January 2013)
Follow-up: ________________________________________________________________

________________________________________________________________________

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Decision: __________________________________________________________________

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__________________________________________________________________________

Marker’s Signature: ______________________________________________________

Principal’s Signature: ____________________________________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________________________________________

Date: ____________________________________________________________________
Table of Questions by Unit and Learning Outcome

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