Transformations and Radical Functions

1. A transformed image of \( y = x^2 \) is shown to the right. If the two graphs are congruent and the vertex of the transformed image is at \((3, -3)\), determine the equation of the transformed image. Explain your answer.

2. As a result of the transformation of the graph of \( y = x^3 \) into the graph of \( y - 4 = (x - 3)^3 \), point \((3, 27)\) becomes point \((6, y)\). The value of \( y \) is __________.

3. If \( y \) is replaced by \( \frac{y}{2} \) in the equation \( y = f(x) \), then the graph of \( y = f(x) \) will be stretched

   A. horizontally about the \( y \)-axis by a factor of \( \frac{1}{2} \)

   B. horizontally about the \( y \)-axis by a factor of 2

   C. vertically about the \( x \)-axis by a factor of \( \frac{1}{2} \)

   D. vertically about the \( x \)-axis by a factor of 2

4. If graph \( g \) is a transformation of graph \( f \), then the equation that generates graph \( g \) is

   A. \( g(x) = f(-x) \)

   B. \( g(x) = f^{-1}(x) \)

   C. \( g(x) = -f(x) \)

   D. \( g(x) = \frac{1}{f(x)} \)

5. Given the graph of a circle with centre at \((0,0)\) and radius 1, as shown to the right, in graph 1, describe the series of transformations required to transform graph 1 to graph 2 which is an ellipse with centre at \((4,5)\), a length of 6 units measured parallel to the \( y \)-axis, and a width of 4 units measured parallel to the \( x \)-axis.
6. The graph \( f(x) = |x+4| + 1 \), is reflected in the \( y \)-axis. This produces the same results as would translating the graph of \( y = f(x) \) to the right by _____ units.

7. The graph of \( y = f(x) = b^x \), where \( b > 1 \), is translated such that the equation of the new graph is expressed as \( y - 2 = f(x - 1) \). The range of the new function is;
   A. \( y > 2 \)
   B. \( y > 3 \)
   C. \( y > -1 \)
   D. \( y > -2 \)

8. Given \( f(x) \), as shown in the graph to the right, the partial graph of \( y = |f(x)| \) is

   \[ A. \quad \begin{array}{c} \text{A.} \end{array} \]
   \[ B. \quad \begin{array}{c} \text{B.} \end{array} \]
   \[ C. \quad \begin{array}{c} \text{C.} \end{array} \]
   \[ D. \quad \begin{array}{c} \text{D.} \end{array} \]

9. If the graph shown to the right is stretched vertically about the line \( x \) axis by a factor of \( \frac{1}{2} \), then it will have;
   A. Invariant points at (0,3) and (4,3)
   B. Invariant points at (-2,0) and (6,0)
   C. An invariant point at (1,4)
   D. An invariant point at (0,3)
10. The graph of \( y = f(x) \), as shown to the right, is transformed by stretching it horizontally about the \( y \)-axis by a factor of \( \frac{1}{2} \), then translating it horizontally left by 1 unit, and finally translating it vertically up by 3 units.

A. Express the resulting transformed function in terms of \( f(x) \).
B. The graph of \( y = f(x) \) is reflected about the \( y \)-axis and then reflected about the \( x \)-axis. State the new \( x \) and \( y \)-intercepts of the transformed graph.
C. State the ordered pairs of any invariant point when \( y = f(x) \) is transformed to \( y = \sqrt{f(x)} \).

11. The graph of a diamond has a centre at point \((0,0)\) and goes through the points \((-1,0), (0,1), (1,0), \) and \((0,-1)\), as shown in the graph above. Describe the series of transformations required to transform the original diamond to the second diamond shown with a centre of \((4,3)\), a vertical length of 6 units measured parallel to the \( y \)-axis, and a horizontal width of 4 units measured parallel to the \( x \)-axis.

12. The graph of \( y = a^x \) and the graph of \( y = \left( \frac{1}{a} \right)^x \), where \( a > 0 \), are reflections of each other about the;
A. \( x \)-axis
B. \( y \)-axis
C. line \( y = x \)
D. line \( y = -x \)

13. To enable larger transport vehicles to pass safely under the arched entry into a park, an engineer was hired to redesign the arch, which rests on 3 m high pillars. The graphs representing the original parabolic arch, \( f(x) \), and a proposed new parabolic arch, \( g(x) \), are shown on the grids to the right. The \( x \)-axis represents ground level.
A. Determine the function of each arch in the form \( y = a(x-h)^2 + k \)
B. Determine the domain of each arch.
C. Determine the range of each arch.
D. Determine the maximum height above the ground for each arch.
E. A student was asked to transform the graph representing the original parabolic arch, \( f(x) \), into the graph representing the proposed parabolic arch, \( g(x) \). The student concluded that the graph of \( f \) would be stretched vertically about the \( x \)-axis by a factor of \( \frac{3}{2} \). Explain why this student is wrong.
14. Three points, A(1,3), B(4,0), C(5,-1) are on the graph of the function \( f \). If the function \( f \) is transformed into the function \( g \), where \( g(x) = 2f(x) + 3 \), then the y-coordinates of the points corresponding to A, B, and C on the graph of the transformed function will be _____, _____, and _____ respectively.

15. The function \( y = f(x) \) is transformed and represented by the dashed line shown in the graph to the right. Which of the following best represents the transformation shown in the graph?
   A. \( y = f(-x) \)
   B. \( -y = f(x) \)
   C. \( x = f(y) \)
   D. \( x = f(-y) \)

16. The parabola \( y = f(x) = -(x-a)(x+b) \) is transformed to \( y = g(x) = \sqrt{f(x)} \). Which of the following is the correct domain of \( y = g(x) \)?
   A. \((-\infty, -a] \cup [b, \infty)\)
   B. \([-a, b]\)
   C. \((-\infty, -b] \cup [a, \infty)\)
   D. \([-b, a]\)

17. A partial graph of \( y = f(x) \) is shown to the right. All of the x-intercepts of \( f \) are shown, and all of the x-intercepts are integers. If \( g(x) = -f(x) \), then the largest x-intercept of the function \( g \) is
   A. 1
   B. 2
   C. 3
   D. 4
18. Three points are shown on the graph of $f$ to the right. If the function, $f$, is transformed into the function $g$, where $g(x) = -f(x-2) + 7$, the the $y$-coordinates of the points corresponding to $A$, $B$, and $C$ on the graph of the transformed function will be _____, _____, and _____ respectively.

19. If $f(x) = x^2 + 3$, then the translation $f(x-4) + 2$ will change
   A. The domain and range of $f$
   B. The domain but not the range of $f$
   C. The range but not the domain of $f$
   D. Neither the domain or range of $f$

20. The graph of $y = f(x)$ is reflected about the $x$-axis, horizontally stretched by a factor of 2 about the $y$-axis, and then translated 3 units to the left and 1 unit down. The equation of the transformed graph is
   A. $y = -f\left(\frac{1}{2}(x-3)\right)-1$
   B. $y = -f\left(\frac{1}{2}(x+3)\right)-1$
   C. $y = -f\left[2(x+3)\right]-1$
   D. $y = -f\left[2(x-3)\right]-1$

21. A quadratic function $y = f(x)$ has the range $(-\infty,5]$ and contains $x$-intercepts at -2 and 6. How many invariant points does this function share with $y = \sqrt{f(x)}$?

22. The function $y = \sqrt{2x+3} - d$ has the same range as the function
   A. $y = -\sqrt{5x+2} - d$
   B. $y = \sqrt{-\frac{x}{3}} - d$
   C. $y = -(\sqrt{2x+3} - d)$
   D. $y = -\sqrt{3x-2} + d$
Additional Transformation/Radical Function Questions from the Exam Bulletin

7. Given \( y - k = a(x - h)^2 \), \( a = 1 \), \( h < 0 \), \( k > 0 \). In which quadrant is the vertex?
   
   A. Quadrant I  
   B. Quadrant II  
   C. Quadrant III  
   D. Quadrant IV

8. Given the functions \( f(x) = |x - 2| + 3 \) and \( g(x) = |x + 2| + 1 \). The transformations that will transform \( y = f(x) \) into \( y = g(x) \) are a translation of:
   
   A. 4 units left and 2 units down  
   B. 4 units right and 2 units up  
   C. 1 unit left and 3 units up  
   D. 2 units left and 4 units down

9. The transformation of the function \( f(x) = x^3 \) is described by the mapping notation \((x, y) \rightarrow (x - 4, y + 9)\). Describe the transformations on \( y = f(x) \).
Use the following information to answer the next question.

The graph of the function \( y = f(x) \) is transformed to produce the graph of the function \( y = g(x) \).

![Graph of f(x) and g(x)](image)

10. An equation for \( g(x) \) in terms of \( f(x) \) is

A. \( g(x) = \frac{1}{2} f(3x) \)
B. \( g(x) = 2 f(3x) \)
C. \( g(x) = \frac{1}{2} f\left(\frac{1}{3}x\right) \)
D. \( g(x) = 2 f\left(\frac{1}{3}x\right) \)

Use the following information to answer the next question.

The graph of \( f(x) = |x| \) and \( y = g(x) \) are shown below. The graph of \( f(x) \) undergoes a single transformation to become the graph of \( g(x) \).

![Graph of f(x) and g(x) for question 11](image)

11. Determine an equation for the function \( g(x) \). (There is more than one correct answer.)
Use the following information to answer the next question.

The graph of \( y = f(x) \) is transformed into the graph of \( g(x) = 2f(x - 3) \). The domain and range of each function are shown below.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph of ( f(x) )</td>
<td>[-1, 3]</td>
</tr>
<tr>
<td>Graph of ( g(x) )</td>
<td>([a, b])</td>
</tr>
</tbody>
</table>

12. For the graph of \( g(x) \), the values of \( a, b, c, \) and \( d \) are, respectively, \( \underline{\quad}, \underline{\quad}, \underline{\quad}, \) and \( \underline{\quad} \).

Use the following information to answer the next question.

The graph of \( y = f(x) \) below is reflected in the y-axis, vertically stretched by a factor of 2 about the x-axis, horizontally stretched by a factor of \( \frac{1}{2} \) about the y-axis, and then translated 3 units up.

13. Sketch the graph of the new function.
Use the following information to answer the next question.

The ordered pairs below represent possible transformations of Point \( P(a, b) \) on the graph of the function \( y = f(x) \).

<table>
<thead>
<tr>
<th>Point 1: ((4a, b))</th>
<th>Point 3: ((a, -b))</th>
<th>Point 5: (\left(\frac{a}{4}, b\right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 2: ((-a, b))</td>
<td>Point 4: (\left(a, \frac{b}{4}\right))</td>
<td>Point 6: ((a, 4b))</td>
</tr>
</tbody>
</table>

14. If \( y = f(x) \) undergoes the following single transformations, identify the coordinates of the corresponding Point \( P \) on the new graph.

The corresponding point on the function \( y = -f(x) \) is point number ________.

The corresponding point on the function \( y = f\left(\frac{1}{4}x\right) \) is point number ________.

The corresponding point on the function \( y = \frac{1}{4}f(x) \) is point number ________.

The corresponding point on the function \( y = f(-x) \) is point number ________.

Use the following information to answer the next question.

The graph of \( y = f(x) \) is reflected in the \( x \)-axis, stretched vertically about the \( x \)-axis by a factor of \( \frac{1}{3} \), and stretched horizontally about the \( y \)-axis by a factor of 4 to create the graph of \( y = g(x) \).

15. For the point \((-3, 6)\) on the graph of \( y = f(x) \), the corresponding point on the graph of \( y = g(x) \) is

A. \((9, 24)\)  
B. \((-12, -18)\)  
C. \((1, 24)\)  
D. \((-12, -2)\)

16. When the graph of \( y = -x^2 + 4 \) is reflected in the \( y \)-axis, the new equation will be

A. \( y = x^2 + 4 \)  
B. \( y = -x^2 + 4 \)  
C. \( y = x^2 - 4 \)  
D. \( y = -x^2 - 4 \)
17. Describe a sequence of transformations required to transform the graph of \( y = \sqrt{x} \) into the graph of \( y = \sqrt{-\frac{1}{2}x - 4 + 10}. \)

**Use the following information to answer the next question.**

The graph of \( y = f(x) \) is shown below.

![Graph of y = f(x)](image)

18. For each transformation of \( y = f(x) \) indicated, the invariant point exists at point number:

\[
\begin{align*}
  y &= -f(x) \\
  y &= f(-x) \\
  x &= f(y)
\end{align*}
\]

19. Verify that \( f(x) = 2x - 3 \) is the inverse of \( g(x) = \frac{1}{2}x + \frac{3}{2} \).
20. A restriction on the domain of \( f(x) = x^2 + 4 \), such that its inverse is also a function, could be:

A. \( \{x \mid x \geq -4\} \)
B. \( \{x \mid x \geq 0\} \)
C. \( \{x \mid x \leq 2\} \)
D. \( \{x \mid x \leq 4\} \)

21. The graph of \( y = 3^x \) is reflected in the line \( y = x \). The equation of the new graph is

A. \( y = \log_3 x \)
B. \( y = 3 \log x \)
C. \( y = 3^{-x} \)
D. \( y = -3^x \)

Use the following information to answer the next question.

The graph of the function \( y = f(x) \) is shown below.

![Graph of y = f(x)](image)

50. Sketch the graph of \( y = \sqrt{f(x)} \) and state the domain and range.

51. State the coordinates of any invariant points when \( f(x) = \frac{1}{2} x - 3 \) is transformed into \( y = \sqrt{f(x)} \).

52. Determine the x-intercept of \( y = -2 \sqrt{x} + 4 + 3 \), to the nearest hundredth, and explain its relationship to the zero of the function.
Exponents and Logarithms

1. A clothing store is going out of business. The owner reduces the cost of each item by 10% of the current price at the start of each week. A jacket costs $120.00 during the 1st week of the sale. If this jacket is still in the store during the 5th week of the sale, then the price of the jacket, to the nearest cent, will be
   A. $70.00
   B. $70.86
   C. $78.73
   D. $80.00

2. Determine the exact value solution for the equation \( \left( \frac{1}{8} \right)^{x-3} = 2(16)^{2x+1} \)

3. If \( \log_x \left( \frac{1}{64} \right) = -\frac{3}{2} \), then \( x \) is equal to
   A. 16
   B. 8
   C. \( \frac{1}{8} \)
   D. \( \frac{1}{16} \)

4. The population of a city was 173 500 on January 1, 1978, and it was 294 000 on January 1, 1992. If the growth rate of the city can be modelled as an exponential function, then the average annual growth rate of the city, expressed to the nearest tenth of a percentage, was
   A. 1.0%
   B. 3.8%
   C. 6.9%
   D. 12.1%
The equation that defines the decibel level for any sound is

\[ L = 10 \log_{10} \left( \frac{I}{I_0} \right) \]

where

- \( L \) = loudness in decibels
- \( I \) = intensity of sound being measured
- \( I_0 \) = intensity of sound at the threshold of hearing

5. Given that normal conversation is 1,000,000 times as intense as \( I_0 \), then the loudness of normal conversation is
   A. 5 decibels
   B. 6 decibels
   C. 16 decibels
   D. 60 decibels

6. The equation \( y = 4^{3x} \) can also be written as
   A. \( y = \frac{\log_3 x}{4} \)
   B. \( y = \frac{\log_4 x}{3} \)
   C. \( x = \frac{\log_3 y}{4} \)
   D. \( x = \frac{\log_4 y}{3} \)

7. Use a graphing method to determine the value of \( x \) if \( 3^x = 4 \).

8. Graph \( y = \log_2 x \), and identify the domain, range, \( x \)- and \( y \)-intercepts, and asymptotes.

9. If \( \log_3 x = 15 \), then \( \log_3 \left( \frac{1}{3} x \right) \) is equal to _____.

10. An investment of $100 is earning 7% interest per annum compounded annually. If the value, \( V \), of the investment after \( t \) years is given by \( V = 100(1.07)^t \), then \( t \), written as a function of \( V \), is:

A. \( t = \frac{\log_{10}(V)}{2} - \log_{10}(1.07) \)

B. \( t = \frac{\log_{10}(V)}{2\log_{10}(1.07)} \)

C. \( t = \log_{10}(V) - 2 - \log_{10}(1.07) \)

D. \( t = \frac{\log_{10}(V) - 2}{\log_{10}(1.07)} \)

11. If \( \log_3 y = c - \log_3 x \), where \( y > 0 \) and \( x > 0 \), then \( y \) is equal to

A. \( c - x \)

B. \( \frac{c}{x} \)

C. \( \frac{c^3}{x} \)

D. \( \frac{3^c}{x} \)

12. Earthquake intensity is given by \( I = I_o(10)^m \), where \( I_o \) is the reference intensity, and \( m \) is magnitude. A particular major earthquake of magnitude 7.9 is 120 times as intense as a particular minor earthquake. What is the magnitude, to the nearest tenth, of the minor earthquake?

13. Solve \( \log_7(x + 1) + \log_7(x - 5) = 1 \), and verify your solution.

14. In the equation \( 3^{2x+1} = 7 \), the value of \( x \) to the nearest hundredth is _____.
15. A student graphed the following four equations;

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
<td></td>
</tr>
<tr>
<td>$y = \log x$</td>
<td>$y = 5^{x-3}$</td>
<td>$y = x - 3$</td>
<td>$y = x$</td>
<td></td>
</tr>
</tbody>
</table>

The solution to $\log_3 x = x - 3$ could be found using;

A. I and II  
B. I and III  
C. II and III  
D. II and IV

16. The table below represents a $1 000 investment that is invested for a full four year cycle. The value of the investment increased by 8%/a compounded annually for the first three years, and then decreases by 11% in the fourth year. This pattern repeats on a four year cycle. Complete the table for year 3 and 4.

<table>
<thead>
<tr>
<th>Year</th>
<th>Initial Value</th>
<th>Final Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1000.00</td>
<td>1080.00</td>
</tr>
<tr>
<td>2</td>
<td>1080.00</td>
<td>1166.40</td>
</tr>
<tr>
<td>3</td>
<td>1166.40</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A. Algebraically show the average compound rate of return of 2.9% over the four year cycle.  
B. A $2500 investment in a different company experienced a loss each year. The value of this investment after $n$ years if modelled by the exponential function $R = 2500(0.97)^n$. Determine, algebraically, the fewest number of years for the initial investment to be half its value.  
C. Explain how a graphical approach could be used to determine the fewest number of years that it would take for the investment in (B) to decrease to less than $900.00. Your explanation must include the function(s) to be graphed, the appropriate window setting and a summary of how the graphs can be used.

17. Sometimes a person learning a new skill will improve quickly at first and then the rate of improvement slows down. An equation that expresses this relationship is $E = 1 - a^{-0.115t}$ where $E$ is the level of expertise attained, $t$ is the learning time calculated in months, and $a$ is a constant. After 8 month of learning a new skill, a student attained a level of expertise of 0.60. The value of $a$, to the nearest tenth, is

A. 0.2  
B. 0.4  
C. 1.7  
D. 2.7
18. The expression \( \log_{\frac{1}{r}} \left( \frac{1}{3} \right) \), \( r > 3 \), is equal to
   A. \( \log_{3} r \)
   B. \( \log_{r} 3 \)
   C. \( \log(3r) \)
   D. \( \log_{3} \left( \frac{1}{r} \right) \)

19. The expression \( \log_{a}(a^{4}b) - \log_{a}(ab) \) is equal to
   A. 3
   B. 4
   C. 3a
   D. \( a^{3} \)

20. The solution to the equation \( 4^{x+2} = 3^{2x} \), correct to the nearest hundredth is
   A. 5.47
   B. 2.55
   C. 0.77
   D. 0.43

21. If \( ab = 24 \), then to the nearest hundredth, the value of \( 2\log a + 2\log b \) where \( a, b > 0 \) is _____.

22. The initial mass of 20 mg of iodine decays to 5 mg after 16 complete days. The half-life of this isotope of iodine is
   A. 120 h
   B. 192 h
   C. 240 h
   D. 384 h

23. The gain, \( G \), measured in decibels of an amplifier is defined by the equation \( G = 10\log \left( \frac{P}{P_{i}} \right) \) where \( P \) is the output power of the amplifier in watts, and \( P_{i} \) is the input power of the amplifier in watts. If the gain of the amplifier is 23 decibels, then the ratio \( \frac{P}{P_{i}} \) is
   A. 10\log(23)
   B. \log(2.3)
   C. \( 10^{23} \)
   D. \( 10^{2.3} \)
24. The value of \( i \) in the compound interested equation \( 5 = (1+i)^6 \) is
   A. \( \sqrt[6]{4} \)  
   B. \( \sqrt[6]{5} - 1 \)  
   C. \( \frac{\log 6}{\log 5} - 1 \)  
   D. \( \frac{\log 5}{\log 6} - 1 \) 

25. If \( \log_2 \cos \theta = -1 \) the smallest value of \( \theta \) must be _____ °.

26. A function is given by \( y = 3 \log_2 x \). If the value of \( x \) doubles, the function \( y = 3 \log_2 x \) is translated _____ units upwards.

27. The intersection point of the asymptotes for the functions \( y = 2 \log_a (x - 4) - 5 \) and \( y = b^{x-4} + 2 \) is \((x, y)\). The value of \( x \) is _____ and the value of \( y \) is _____.

28. For the equation \( 96 = ab^{\log_2 8} \), the value of \( a \) must be _____.
Additional Exponents and Logs  Questions from the Exam Bulletin

22. The value of $\log_5 625 + 3 \log_7 49 + \log_2 \frac{1}{16} + \log_p b + \log_a 1$ is __________.

23. The equation $m \log_p n + 5 = q$ can be written in exponential form as
   
   A. $p^{(q-5)} = mn$
   B. $p^{(q-5)} = n^m$
   C. $p^{(q-5)} = \frac{m}{n}$
   D. $p^{(q-5)} = m^n$

24. Rank these logarithms in order from least to greatest: $\log_4 62$, $\log_6 36$, $\log_3 10$, $\log_5 20$.

25. The expression $(3^{\log x})(3^{\log x})$ is equivalent to
   
   A. $3^{\log x^2}$
   B. $9^{\log x^2}$
   C. $3^{(\log x)^2}$
   D. $9^{(\log x)^2}$

26. Written as a single logarithm, $2 \log x - \frac{\log z}{2} + 3 \log y$ is
   
   A. $\log \left( \frac{x^2 y^3}{\sqrt{z}} \right)$
   B. $3 \log \left( \frac{xy}{z} \right)$
   C. $\log \left( \frac{x^2}{y^3 \sqrt{z}} \right)$
   D. $\log \left( x^2 - \sqrt{z} + y^3 \right)$
27. Given that $\log_3 a = 6$ and $\log_3 b = 5$, determine the value of $\log_3(9ab^2)$.

Use the following information to answer the next question.

A student’s work to simplify a logarithmic expression is shown below, where $a > 1$.

**Step 1**  
$2\log_a x^4 - 3\log_a x^2 + 4\log_a x^3$

**Step 2**  
$\log_a x^8 - \log_a x^6 + \log_a x^{12}$

**Step 3**  
$\log_a \left( \frac{x^8}{x^6 \times x^{12}} \right)$

**Step 4**  
$\log_a \left( \frac{x^8}{x^{18}} \right)$

**Step 5**  
$\log_a x^{10}$

28. The student’s first recorded error is in Step

A. 2
B. 3
C. 4
D. 5
29. The equation of the asymptote for the graph of \( y = \log_b(x - 3) + 2 \), where \( b > 0 \) and \( b \neq 1 \), is

A. \( y = 2 \)
B. \( y = -2 \)
C. \( x = 3 \)
D. \( x = -3 \)

Use the following information to answer the next question.

A student sketched the graphs of \( f(x) = \log_a(x + 3) - 7 \) and \( g(x) = a^{(x-2)} + 5 \), where \( a > 1 \), on a coordinate plane. She also drew the asymptotes of the two graphs using dotted lines.

30. The intersection point of the two dotted lines will be at

A. \((3, 5)\)
B. \((-3, 5)\)
C. \((2, -7)\)
D. \((-2, -7)\)

31. For the graph of \( y = \log_b(3x + 12) \), where \( 0 < b < 1 \), the domain is

A. \( x > -4 \)
B. \( x > 4 \)
C. \( x > -12 \)
D. \( x > 12 \)

32. The \( y \)-intercept on the graph of \( f(x) = a^{(x+1)} + b \) is

A. \( a \)
B. \( b \)
C. \( 1 + b \)
D. \( a + b \)

33. Algebraically solve the equation \( 8^{(3x+4)} = 4^{(x-9)} \).
34. Solve the equation $3^{(2x+1)} = \left(\frac{1}{5}\right)^{x-3}$ algebraically. Round to the nearest hundredth, if necessary.

35. Solve algebraically $\log_7(x + 1) + \log_7(x - 5) = 1$.

Use the following information to answer the next question.

Earthquake intensity is given by $I = I_0 \times 10^M$, where $I_0$ is the reference intensity and $M$ is the magnitude. An earthquake measuring 5.3 on the Richter scale is 125 times more intense than a second earthquake.

36. Determine, to the nearest tenth, the Richter scale measure of the second earthquake.

37. The population of a particular town on July 1, 2011 was 20 000. If the population decreases at an average annual rate of 1.4%, how long will it take for the population to reach 15 300?

38. Jordan needs $6 000 to take his family on a trip. He is able to make an investment which offers an interest rate of 8% per annum compounded semi-annually. How much should Jordan invest now, to the nearest dollar, so that he has enough money to go on a family trip in 3 years?

### Trigonometry

1. Convert the angles $\frac{2\pi}{3}$ and 1.6 rad to degrees, correct to the nearest tenth.

2. In one minute, the second hand of a clock completes one revolution around the clock face. In $1\frac{1}{2}$ minutes, the second hand of a clock completes an angle of

   A. $\frac{3\pi}{2}$
   B. $3\pi$
   C. $6\pi$
   D. $180\pi$
3. If the circumference of Earth at the equator is approximately 40 070 km, then the shortest arc length from point A to point B, correct to the nearest kilometre, is
   A. 31 026 km
   B. 16 474 km
   C. 8 237 km
   D. 4 938 km

4. Given that \( \sin x = 0.5395 \), and that \( x \) is an acute angle,
   A. what is \( x \) in radians, to the nearest hundredth?
   B. Given that \( \sin x = 0.5395 \) and that \( 0 \leq x < 2\pi \), are there any additional solutions? If so, find them and explain why they are also solutions.
   C. Explain why \( \sin x = 1.5395 \) has no solution for \( x \).

5. Given an equilateral triangle with a side of 2 units, as shown to the right, determine the exact value of \( h \).

6. Determine the solutions to the following trigonometric equations.
   A. \( 1 + 2\cos x = 5\cos x \), where \( 0 \leq x < 2\pi \). Give solutions in decimal form, correct to the nearest hundredth.
   B. \( \cos 4x = 0.5 \), where \( 0 \leq x < 2\pi \). Give solutions correct to the nearest hundredth.

7. Explain why \( \tan \frac{\pi}{2} \) is undefined. Are there any of the remaining five trigonometric ratios that have undefined values? Explain why.
8. When the following pairs of functions are graphed, the pair that could not be used to solve the equation \(4\sin x - 1 = 0\) is;
   A. \(y = \sin x\) and \(y = 1\)
   B. \(y = \sin x\) and \(y = \frac{1}{4}\)
   C. \(y = 4\sin x\) and \(y = 1\)
   D. \(y = 4\sin x - 1\) and \(y = 0\)

9. Graph \(y = 6\sin^2 A - \sin A - 1\), using the window on your graphing calculator.
   A. Predict the number of solutions to \(6\sin^2 A - \sin A - 1 = 0\), \(0 \leq A < 2\pi\), from your graph and and give a reason for your prediction.
   B. Using your calculator, estimate the solutions of \(6\sin^2 A - \sin A - 1 = 0\), \(0 \leq A < 2\pi\), to the nearest tenth.
   C. Confirm your estimates algebraically by finding \(A\), correct to the nearest hundredth for \(0 \leq A < 2\pi\)

10. Determine the general solution of \((2\sin A - 1)(\tan A - 1) = 0\).

11. Use technology to graph \(y = \frac{x}{2} - 2\sin 3x\) in radian mode, and then
   A. determine the total number of \(x\)-intercepts over the real numbers
   B. determine the lowest possible value of \(x\), to the nearest tenth, for \(\frac{x}{2} - 2\sin 3x = 0\).
   C. explain how a student could answer part a above by entering \(y = \frac{x}{2}\) and \(y = 2\sin 3x\) on a graphing calculator?

12. To create an identity (a statement that is true for all \(x\) in the domain) for the equation \(\cos^2 x(1 + \cot^2 x) = A\), the value of \(A\) would need to be;
   A. \(\sin^2 x\)
   B. \(\cos^2 x\)
   C. \(\cot^2 x\)
   D. \(\sec^2 x\)

13. Prove algebraically that \(\sin x \cot x = \cos x\), \(\sin x \neq 0\). And, given that \(x\) is an acute angle, determine \(x\), correct to the nearest degree, such that \(\sin x \cot x = 0.4\)
14. Given the identity \( \frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x} \), \( \cos x \neq 1, \sin x \neq 0 \),
   
   A. verify the identity for the particular case \( x = \frac{\pi}{3} \)
   
   B. demonstrate by graphing \( y = \frac{\sin x}{1 - \cos x} \) and \( y = \frac{1 + \cos x}{\sin x} \), how the graphs verify the identity
   
   C. prove the identity for a general angle, using an algebraic approach
   
   D. identify the values of \( x \) for which this identity is undefined

Refraction describes the bending of light rays. Refraction can be calculated using the formula \( n = \frac{\sin(\theta + \alpha)}{\sin \theta} \)

15. If \( \alpha = 30^\circ \), then an equivalent expression for \( n \) is
   
   A. \( \frac{\sqrt{3}}{2} + \cos \theta \)
   
   B. \( \frac{\sqrt{3}}{2} + \frac{1}{2} \cot \theta \)
   
   C. \( \frac{\sqrt{3}}{2} + \frac{1}{2} \cos \theta \)
   
   D. \( \frac{1}{2} + \frac{\sqrt{3}}{2} \cot \theta \)

16. If the terminal arm of angle \( x \), in standard position, passes through point \((-b, 2b)\), where \( b > 0 \), then the exact values of \( \sin x \), \( \cos x \), and \( \tan x \) are, respectively;
   
   A. \( \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 2 \)
   
   B. \( \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -2 \)
   
   C. \( -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, -2 \)
   
   D. \( \frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 2 \)

17. Given that \( \cos x = -\frac{7}{8} \) and \( \tan x > 0 \), determine the exact value of \( \sin x \).
18. The three solutions of the equation \( f(x) = 0 \) are 0°, 180°, and 360°. Therefore, the three solutions of the equation \( f(x - 30°) = 0 \) are
A. 0°, 60°, and 120°
B. –30°, 150°, and 330°
C. 30°, 210°, and 390°
D. 0°, 540°, and 1 080°

19. Which of the following statements does not describe the graph of \( f(x) = -3\sin\left(x - \frac{\pi}{2}\right) \)?
A. The amplitude is 3.
B. The period is \( 2\pi \).
C. The graph of \( f(x) = -3\sin\left(x - \frac{\pi}{2}\right) \) is the same as the graph of \( f(x) = -3\sin x \) with a phase shift of \( \frac{\pi}{2} \) to the right.
D. The graph of is the same as the graph of \( f(x) = -3\sin\left(x - \frac{\pi}{2}\right) \) with a vertical translation of 3 units down.

The distance above the ground of a passenger on a circular ferris wheel is given by the equation

\[
h(t) = 5\sin\left(\frac{\pi}{12}(t - 6)\right) + 6
\]

where \( h \) is the distance above the ground, in metres, and \( t \) is the time, in seconds, after the passenger passes the lowest point of the ride for the first time.

20. The distance of the passenger above the ground 10 s after passing the lowest point of the ride, to the nearest tenth of a metre, is
A. 4.6 m
B. 6.1 m
C. 8.5 m
D. 10.3 m
The graph to the right shows the height, \( h \), in metres above the ground, over time, \( t \), in seconds that it takes a ferris wheel to make one complete revolution of the ride. The maximum height of the ferris wheel is 19 m and the minimum height is 1 m.

21. What is the period for 1 revolution of the ferris wheel?
   A. How high is the hub or centre of the ferris wheel off the ground?
   B. Write an equation for the height, \( h \), of the ferris wheel as a function of time, \( t \).
      Use the sine function for \( h \) in terms of \( t \).
   C. Find the distance from the ground, to the nearest tenth of a metre, of a particular point on the ride at \( t = 10 \text{ s} \).
   D. Using technology, find the first time, to the nearest tenth of a second, that a particular point on the ferris wheel is 6 m above the ground.

22. Given \( \cos 2x = \frac{1}{\sqrt{2}} \), where \( 0 \leq x < 2\pi \), find the approximate values \( x \) correct to the nearest hundredth.

23. Determine the domain, range, and period of \( f(x) = \cot x \) and \( g(x) = \cot (2x) \).

24. Determine the exact value of \( \sin^2\left(\frac{7\pi}{6}\right) + \tan^2\left(\frac{\pi}{3}\right) \).

25. If \( \cos \theta - k = 0 \), and \( \theta = -120^\circ \), then the value of \( k \) must be;
   A. \( \frac{1}{2} \)
   B. \( -\frac{1}{2} \)
   C. \( -\frac{\sqrt{3}}{2} \)
   D. \( \frac{\sqrt{3}}{2} \)

26. Graphically determine the solutions to the equations;
   A. \( \sin(2x) = \frac{-\sqrt{3}}{2} \), \( 0^\circ \leq x < 360^\circ \)
   B. \( \cot x = 1.4 \), \( 0 \leq x < 2\pi \), correct to the nearest hundredth of a radian.

27. Given \( \csc \theta = \frac{5}{3} \), \( 0 < \theta < \frac{\pi}{2} \), determine the five other trigonometric ratios.
28. The expression \( \frac{\sin x + \cos x}{\csc x + \sec x} \) is equal to;
   A. \( \sin x \cos x \)
   B. \( \tan^2 x \sin x \)
   C. \( \sec x \csc x \)
   D. \( \sin^2 x + \cos^2 x \)

29. Given \( f(x) = \frac{1 - \cos^2 x}{\tan x} \);
   A. Graph \( y = f(x) \) using the window settings \( x: [-2\pi, 2\pi, \frac{\pi}{2}] \), \( y: [-1, 1, 0.5] \)
   B. State the period and amplitude of this function
   C. Find the values of \( x \) for which the function is undefined
   D. State a single trigonometric function that is equivalent to \( f(x) = \frac{1 - \cos^2 x}{\tan x} \).

30. The following question is based on the function shown to the right, with a minimum point \( \left(0, \frac{1}{2}\right) \) and a maximum point at \( \left(\frac{\pi}{3}, \frac{3}{2}\right) \).
   A. Identify the amplitude and period.
   B. Write a function in terms of \( \sin \).
   C. Write a function in terms of \( \cos \).

31. The partial graph of \( f(x) = \sec x \) and the partial graph of \( g(x) = a\sec x \) are shown to the right. Determine the value of \( a \), to the nearest whole number.

32. An equation equivalent to the equation \( \tan^2 x + \sec x - 1 = 0 \) is;
   A. \( \sec^2 x + \sec x - 2 = 0 \)
   B. \( \sec^2 x - \sec x - 2 = 0 \)
   C. \( \sec^2 x + \sec x = 0 \)
   D. \( 3\sec x - 2 = 0 \)
33. To verify the identity \( \sin x \tan x + \sec x = \frac{\sin^2 x + 1}{\cos x} \), Sam substituted 1.4 rad for \( x \) on each side. Sam found that each side of the identity, correct to the nearest tenth, is equal to _____.

34. If the equation \( \sin^2 x + \sin x - 2 = 0 \) where \( 0^\circ \leq x < 360^\circ \), is satisfied by \( x = 90^\circ \), then the equation \( \sin^2(3x) + \sin(3x) - 2 = 0 \), where \( 0^\circ \leq x < 360^\circ \) is satisfied by;
   A. Only the solution \( x = 90^\circ \)
   B. Only the solution \( x = 270^\circ \)
   C. Only the solutions \( x = 30^\circ, 120^\circ, 210^\circ \)
   D. Only the solutions \( x = 30^\circ, 150^\circ, 270^\circ \)

35. In a region of Canada, biologists have studied the population of rabbits over a 20 year period. The found the population, \( N \), varied as a function of time \( t \), in years, by \( N(t) = -1500\sin(0.8t) + 2000 \). Given this model, the maximum number of rabbits that were present at any time in this 20 year period was;
   A. 3500 rabbits
   B. 2000 rabbits
   C. 1500 rabbits
   D. 500 rabbits

36. The equation \( y = \sin\left(\frac{\pi}{2} x\right) + 7 \) was graphed. The viewing window settings are \( x: [5, x_{\text{max}}, 1] \) and \( y: [6, 8, 1] \). If one period of the graph is showing, the value of \( x_{\text{max}} \) must be _____.
Biorhythm theory states that all humans are influenced by three biological cycles that begin at birth and continue throughout our lives. Each cycle has a different period: Physical ($P$) 23 days, Emotional ($E$) 28 days, and Intellectual ($I$) 33 days. They are graphed as sinusoidal curves and shown below, where $L$ is the level of the cycle and $t$ is the time in days. The level of each cycle, $L_P$, $L_E$ and $L_I$ has a maximum peak of 100% (positive peak) and a minimum value of -100% (negative peak).

A. Identify one of the parameters, $a$, $b$, $c$ or $d$ that would be equal in all three graphs. Justify.

B. Which cycle has a graph that would be considered to have horizontal phase shift of 16 days if its equation were written as a sine function?

C. The “biorhythm index” is the sum of all the levels of the three cycles on any given day. For the days on which the Intellectual is at 50%, what day in the first 28 day period will the biorhythm index be highest?

D. The graph of the Physical could be modelled by a cosine function that has its positive peak at $t = 8.75$ days. Write an equation of the form $L_P = a \cos(b(t-c)) + d$ that describes the graph of the Physical.

E. The Emotional for a different person has a graph described by $L_E = 100 \sin\left(\frac{2\pi}{28} (t-7)\right)$. Graph the equation and sketch one full period indicating the maximum points, minimum points and the intercepts.

F. Determine, to the nearest hundredth of a percentage, the level of Emotional for this person (from E) on day 9.
38. If the general solution to the equation \( \tan(x) = 1 \) is \( x = \frac{\pi}{4} + n\pi, \ n \in I \), then the general solution to the equation \( \tan(3x) = 1 \) is

A. \( \frac{\pi}{4} + n\pi, \ n \in I \)
B. \( \frac{3\pi}{4} + 3n\pi, \ n \in I \)
C. \( \frac{\pi}{12} + n\pi, \ n \in I \)
D. \( \frac{\pi}{12} + \frac{n\pi}{3}, \ n \in I \)

39. When the graph of \( f(\theta) = \sin \theta \) is transformed to the graph of \( g(\theta) = k \sin \theta, \ k > 1 \), it will undergo a change in its

A. Range
B. Domain
C. y-intercept
D. \( \theta \)-intercepts

40. \( g(x) = 1 - \cos^2 x \), where \( x \) is in radians.

A. Graph \( y = g(x) \) on your calculator, and state the period and range of the sinusoidal graph.
B. The graph of function \( g \) can also be described by \( h(x) = a \cos \left[ b \left( x - \frac{\pi}{2} \right) \right] + d \). State the values of \( a, b, \) and \( d \), and write the equation of \( h(x) \).
C. Rewrite the function \( h \) as a sine function in the form \( j(x) = a \sin \left[ b(x - c) \right] + d \).
D. Algebraically prove the identity \( \cos^2 x \tan^2 x = 1 - \cos^2 x \).
E. Graph \( f(x) = \cos^2 x \tan^2 x \) on your calculator, and compare this graph with the graph of \( g(x) = 1 - \cos^2 x \). Although the graph of \( f \) and the graph of \( g \) appear to be identical on the graphing calculator for all values of \( x \), they are not. Explain the difference that exists between the two functions.

41. If \( \sin \theta = 0.6 \) and \( \cos \theta = 0.8 \) then the value of \( \tan \theta \), to the nearest hundredth is _____.

42. If \( \cot \theta = \sqrt{3} \), where \( 0 < \theta < \frac{\pi}{2} \) then the exact value of \( \cos \theta \) is

A. \( \frac{1}{2} \)
B. \( \frac{1}{\sqrt{3}} \)
C. \( \frac{\sqrt{3}}{2} \)
D. \( \frac{2}{\sqrt{3}} \)
43. If the general solution to the equation \( \tan x = 1 \) is \( x = \frac{\pi}{4} + n\pi, n \in I \) then the general solution to the equation \( \tan(3x) = 1 \) is
   A. \( \frac{\pi}{12} + n\pi, n \in I \)
   B. \( \frac{3\pi}{4} + 3n\pi, n \in I \)
   C. \( \frac{\pi}{12} + n\pi, n \in I \)
   D. \( \frac{\pi}{3} + n\pi, n \in I \)

44. A mass that is suspended on a spring is at rest, 1.0 m above a table. A student pulls the mass downward and releases it setting it in motion. If it is assumed that there is no friction, the height of the mass relative to the table can be modelled by the equation \( h(t) = -0.3\cos(\pi t) + 1 \) where \( h(t) \) is the height of the mass, in meters, above the table at time \( t \) seconds. How many seconds will it take the mass to reach a height of 1.2 m above the table for the first time?
   A. 0.73 s
   B. 1.00 s
   C. 1.24 s
   D. 1.27 s

45. The expression \( \sqrt{\frac{1 - \cos^2 x}{1 + \cot^2 x}} \) is equal to
   A. \( \frac{\sin x(1 - \cos x)}{\sin x - \cos x} \)
   B. \( \sin^2 x \)
   C. \( \sin x \)
   D. 1

46. The terminal arm of angle \( x \), in standard position, passes through the point \( P(-2, 3) \). A value of \( x \), to the nearest tenth of a degree is
   A. 33.7
   B. 56.3
   C. 123.7
   D. 146.3

47. Which of the following statements identifies the period and the asymptotes or the period and the range of the graph of \( y = \csc 2\theta \)?
   A. The graph of \( y = \csc 2\theta \) has a period of \( \pi \) and asymptotes of \( x = \frac{n\pi}{2}, n \in I \)
   B. The graph of \( y = \csc 2\theta \) has a period of \( \frac{\pi}{2} \) and asymptotes of \( x = n\pi, n \in I \)
   C. The graph of \( y = \csc 2\theta \) has a period of \( \frac{\pi}{2} \) and a range of \( y \leq -1 \) or \( y \geq 1 \)
   D. The graph of \( y = \csc 2\theta \) has a period of \( \pi \) and a range of \( y \leq -2 \) or \( y \geq 2 \)
48. The exact value of \( \cos 15^\circ \) is equal to
A. \( \cos 45^\circ - \cos 30^\circ \)
B. \( \cos 75^\circ - \cos 60^\circ \)
C. \( \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \)
D. \( \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \)

49. When the graph of \( f(\theta) = \sin \theta \) is transformed to the graph of \( g(\theta) = k \sin \theta, k > 1 \), it will undergo a change in its
A. Range  
B. Domain  
C. y-intercepts  
D. \( \theta \)-intercepts

50. A bench is designed based on the partial arc length of a circle, as shown to the right. If the inside arc of the bench is designed by creating an arc from a circle of radius 1.4 m, and an angle of 72\(^\circ\), and the width of the bench seat is 0.5 m, what is the perimeter distance around this bench correct to the nearest tenth of a metre?

51. If \( \frac{3(\sin x + \cos x)^2 - 3}{\sin^2 x} = k \cot x \), then the value of \( k \) must be ______.

52. The point \( \left( \frac{\sqrt{3}}{2}, -\frac{1}{2} \right) \) is on the terminal arm of angle \( x \). If \(-6\pi < x < -4\pi \), then the value of \( x \) in radians is \( -\frac{a\pi}{b} \). The value of \( a \) is _____ and the value of \( b \) is _____.

53. For the equation \( \cot^2 x + \csc^2 x = 1 \), the general solutions could be represented by \( x = \frac{\pi}{a} + b\pi n, \quad n \in \mathbb{I} \). The value of \( a \) is _____ and the value of \( b \) is _____.

54. If the function \( y = 2\sin(2x - 1) + 4 \) contains the point \((x, 5)\), correct to the nearest tenth, the correct value of \( x \), \( 0 \leq x < 90^\circ \) must be _____.
Additional Trigonometry Questions from the Exam Bulletin

1. The angle $\frac{15\pi}{4}$, converted to degrees, is $\underline{525^\circ}$.

2. An angle, in radians, that is co-terminal with $30^\circ$ is
   
   A. $-\frac{5\pi}{6}$
   
   B. $\frac{13\pi}{6}$
   
   C. $\frac{7\pi}{6}$
   
   D. $\frac{25\pi}{6}$

   Use the following information to answer the next question.

   An angle, $\theta$, in standard position, is shown below.

   ![Diagram of an angle in standard position]

3. The best estimate of the rotation angle $\theta$ is
   
   A. 1.25 radians
   
   B. 3.12 radians
   
   C. 4.01 radians
   
   D. 5.38 radians
Mary is given the diagram below, showing an angle rotation of 120°. The arc length of the sector is 40 cm.

Statement 1  The radius of the circle, to the nearest centimetre, is 19 cm.
Statement 2  An equivalent angle rotation is $\frac{4\pi}{3}$.
Statement 3  If the arc length on this circle increases to 80 cm, then the central angle must be 240°.
Statement 4  Mary can determine the radius of the circle by dividing the given angle by the arc length.

4. The two statements above that are correct are numbered _____ and _____.

Use the following information to answer the next question.
Use the following information to answer the next question.

A circle with a radius, \( r \), an arc length of \( 34\pi \), and two central angles of \( \frac{a\pi}{15} \) and \( \frac{17\pi}{15} \) is shown below.

The value of \( a \) in the angle \( \frac{a\pi}{15} \) is \( bc \).

The length of the radius, \( r \), of the circle, to the nearest whole number, is \( de \).

5. The values of \( b, c, d, \) and \( e \) are, respectively, _____, _____, _____, and _____.

Use the following information to answer the next question.

If the point \( P(0,2, k) \) lies on a circle with centre at the origin and a radius of 1, then the exact value of \( k \) can be expressed as \( \pm \sqrt{b} \).

6. The value of \( b \), to the nearest hundredth, is _________.

7. On a unit circle, if Point \( P\left(-\frac{5}{13}, \frac{12}{13}\right) \) lies on the terminal arm of an angle in standard position, what are the exact values of the 6 trigonometric ratios?

8. Given that \( \csc \theta = -\frac{8}{5} \), where \( \pi < \theta < \frac{3\pi}{2} \), determine the exact value of \( \tan \theta \).

9. Determine the exact value of \( \sin\left(-\frac{\pi}{6}\right) + \cos\left(\frac{7\pi}{4}\right) \).

10. If \( \tan \theta = \frac{5}{2} \), where \( 0 \leq \theta \leq 2\pi \), then the largest positive value of \( \theta \), to the nearest tenth, is _________. rad.
Use the following information to answer the next question.

Point $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and point $B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lie on the terminal arm of two different angles in standard position. The angle, $\theta$, where $0 < \theta < \pi$, can be expressed in the form $\frac{a\pi}{b}$.

11. The values of $a$ and $b$ are, respectively, _____ and _____.

Use the following information to answer the next question.

Each of the trigonometric ratios listed below results in a value of zero, or is undefined.

\[
\begin{align*}
\tan\left(\frac{\pi}{2}\right) \\
cot\left(\frac{3\pi}{2}\right) \\
\sin\pi \\
csc(2\pi)
\end{align*}
\]

12. Use the following code to indicate that the value of the ratio is zero, or that the ratio is undefined.

1 = The value of the ratio is zero.
2 = The ratio is undefined.

<table>
<thead>
<tr>
<th>Ratio:</th>
<th>$\tan\left(\frac{\pi}{2}\right)$</th>
<th>$\cot\left(\frac{3\pi}{2}\right)$</th>
<th>$\sin\pi$</th>
<th>$\csc(2\pi)$</th>
</tr>
</thead>
</table>
Use the following information to answer the next question.

For the angles \( \frac{\pi}{6} \), \( \frac{5\pi}{6} \), \( \frac{7\pi}{6} \), \( \frac{11\pi}{6} \), the following statements are given.

**Statement 1**  They all have the same reference angle.

**Statement 2**  These angles in degrees are, respectively, 30°, 150°, 210°, and 300°.

**Statement 3**  They are all part of the solution set \( \theta = \frac{\pi}{6} + 2n\pi \), \( n \in \mathbb{Z} \).

**Statement 4**  The values of \( \sin \left( \frac{\pi}{6} \right) \) and \( \sin \left( \frac{5\pi}{6} \right) \) are positive.

13. The two statements that are true from the list above are numbered _____ and _____.

14. For the function \( y = a \cos \theta + d \), the range is \([-4, 10]\). What are the values of \( a \) and \( d \)?

15. For the function \( y = \sin(3x + \pi) + 7 \), what is the phase shift and period of the corresponding graph?

16. Given that \( f(\theta) = \cos(n\theta) \) has the same period as the graph of \( g(\theta) = \tan \theta \), the value of \( n \) is ____________.

Use the following information to answer the next question.

The partial graph of the cosine function below has a minimum point at \( \left( \frac{\pi}{2}, -2 \right) \) and a maximum point at \( (\pi, 8) \). The equation of the function can be expressed in the form \( y = a \cos(bx) + d \).

![](image)

17. What are the values of \( a \), \( b \), and \( d \)?
Use the following information to answer the next question.

For the graph of the function \( f(x) = -3 \sin[2(x - 5)] + d \) the following statements were made.

**Statement 1**  The amplitude is 3.

**Statement 2**  The maximum value is \((d - 3)\).

**Statement 3**  The period is \(2\pi\).

**Statement 4**  When compared to the graph of \( g(x) = -3 \sin(2x) + d \), the graph of \( y = f(x) \) has been horizontally translated 5 units to the right.

**Statement 5**  If \( d > 3 \), then the graph of \( y = f(x) \) will have no \( x \)-intercepts.

18. The number of true statements about the graph of \( y = f(x) \) from the list above is

A. 1  
B. 2  
C. 3  
D. 4

Use the following information to answer the next question.

The height of a point on a Ferris wheel, \( h \), in metres above the ground, as a function of time, \( t \), in seconds can be represented by a sinusoidal function. The maximum height of the Ferris wheel above the ground is 17 m and the minimum height is 1 m. It takes the Ferris wheel 60 seconds to complete two full rotations.

19. If the particular point starts at the minimum height above the ground, then write an equation for the height of this point on the Ferris wheel, \( h \), as a function of time, \( t \), in the form \( h = a \cos[b(t - c)] + d \).

20. Solve for \( \theta \), where \( 180^\circ \leq \theta < 360^\circ \), in the equation \( 2 \cos^2 \theta + \cos \theta = 0 \).

21. Determine a general solution of \( \tan^2 \theta - 1 = 0 \), expressed in radians.

22. For the equation \( 2 \cos^2 x + \sin x - 1 = 0 \), find all values of \( x \), where \(-\pi \leq x \leq \pi\).

23. Graphically solve for \( \theta \), where \(-180^\circ \leq \theta \leq 0^\circ\), given \( (2 - \sqrt{3} \sec \theta)(\sec \theta + 3) = 0 \). State answers to the nearest degree.
A Mathematics 30–1 class was asked to determine a general solution to the equation 
\[ \sin(2\theta) - \cos \theta = 0, \] in degrees. The answers provided by four different students are shown below.

<table>
<thead>
<tr>
<th>Student</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>[ \theta = 60^\circ + n(120^\circ), \quad n \in \mathbb{Z} ]</td>
</tr>
<tr>
<td>Student 2</td>
<td>[ \theta = 90^\circ + n(360^\circ), \quad n \in \mathbb{Z}, \quad \text{and} \quad \theta = 30^\circ + n(120^\circ), \quad n \in \mathbb{Z} ]</td>
</tr>
<tr>
<td>Student 3</td>
<td>[ \theta = n(180^\circ), \quad \theta = 60^\circ + n(360^\circ), \quad \text{and} \quad \theta = 300^\circ + n(360^\circ), \quad n \in \mathbb{Z} ]</td>
</tr>
<tr>
<td>Student 4</td>
<td>[ \theta = 90^\circ + n(180^\circ), \quad \theta = 30^\circ + n(360^\circ), \quad \text{and} \quad \theta = 150^\circ + n(360^\circ), \quad n \in \mathbb{Z} ]</td>
</tr>
</tbody>
</table>

24. The two students who provided a correct general solution are numbered
   
   A. 1 and 3
   B. 1 and 4
   C. 2 and 3
   D. 2 and 4

25. Three students were given the identity 
   \[ \frac{\sin^2 \theta - 1}{\cos \theta} = -\cos \theta, \quad \text{where} \quad \cos \theta \neq 0. \]

   a) Student A substituted in \[ \theta = \frac{\pi}{3} \] to both sides of the equation and got \( LS = RS \). Student B entered \( LS \) into \( y_1 \) and \( RS \) into \( y_2 \) and concluded that the graphs are exactly the same. Explain why these methods are not considered a proof of this identity.

   b) Student C correctly completed an algebraic process to show \( LS = RS \). Show a process Student C might have used.

   c) Which non-permissible values of \( \theta \) should be stated for this identity?

26. The expression \[ \frac{\cot x + \csc x}{\sec x + 1}, \quad \text{where} \quad \sec x \neq -1, \] is equivalent to

   A. \( \sin x \)
   B. \( \tan x \)
   C. \( \csc x \)
   D. \( \cot x \)
Use the following information to answer the next question.

Each trigonometric expression below can be simplified to a single numerical value.

1. \( \cot^2 x - \csc^2 x \)
2. \( \sec^2 x - \tan^2 x \)
3. \( \sin x \frac{\tan x}{\sec x} \)
4. \( \frac{1}{7} \cos^2 x + \frac{1}{3} \sin^2 x \)

27. When the numerical values of the simplified expressions are arranged in ascending order, the expression numbers are _____, _____, _____, and _____.

28. What is the exact value of \( \tan 75^\circ \)?

29. Prove algebraically that \( \frac{2 \tan x}{1 - \tan^2 x} = \frac{\sin(2x)}{\cos^2 x - \sin^2 x} \), where \( x \neq \frac{\pi}{4} + \frac{n\pi}{2}, n \in \mathbb{Z} \).

30. Given that \( \sin \theta = -\frac{2}{7} \), and \( \cot \theta < 0 \), determine the exact value of \( \cos \left( \theta - \frac{2\pi}{3} \right) \).
Permutations and Combinations

A paperboy who delivers papers on his bike can travel only on the trails represented in the diagram below.

1. The number of different trails that the paperboy can take to get from house B to house A without backtracking is
   A. 13  
   B. 32  
   C. 60  
   D. 72

2. In a particular town, all of the streets run north–south or east–west. A student lives 5 blocks west and 3 blocks south of a school. The number of different routes, 8 blocks in length, that the student can take to get to the school is __________.

3. An airline pilot reported that in seven consecutive days she spent, in an unspecified order, one day in Winnipeg, one day in Regina, two days in Edmonton, and three days in Yellowknife. How many different itineraries are possible? How many itineraries were possible if she spent the first day and the last day in Yellowknife?

4. If all of the letters in the word DIPLOMA are used, then the number of different 7-letter arrangements that can be made beginning with 3 vowels is _____.

5. A school committee consists of 1 vice-principal, 2 teachers and 3 students. The number of different committees that can be selected from 2 vice-principals, 5 teachers, and 9 students is
   A. 20 160  
   B. 8 008  
   C. 1 680  
   D. 90
6. How many diagonals are there in a regular polygon with 20 sides? What is the general formula for the number of diagonals in an \( n \)-sided polygon?

7. A term of the binomial expansion \((ax + y)^8\), where \(a > 0\), is \(112x^2y^6\). The value of \(a\), correct to the nearest whole number, is __________.

8. Solve \( \binom{n}{2} = \frac{nP_3}{3!} \), and verify your solution.

9. In a group of people, there are 10 females and 12 males. Determine the number of 4-member committees consisting of at least 1 female that can be formed.

10. Determine the number of possible paths possible from point \( A \) to point \( B \) in the diagram to the right, if travel may occur only along the edges of the cubes and if the path must always move closer to \( B \).

11. A teacher tells his students that on a multiple-choice test of 12 questions, 3 answers are \( A \), 3 are \( B \), 3 are \( C \), and 3 are \( D \). How many different answer keys are possible?

12. Expand \( \left( 2a - \frac{3}{b} \right)^4 \) by using the binomial theorem.

13. Determine the constant term in the expansion of \( \left( 2x^2 - \frac{1}{x} \right)^6 \).

14. Investigate the sample space for flipping 1 coin, 2 coins, 3 coins, 4 coins, and \( n \) coins, and make an organized list of outcomes. Relate this organized list to Pascal’s triangle and the binomial theorem.

15. A car manager wants to line up 10 cars of identical model except for colour. There are 3 red cars, 2 blue cars and 5 green cars. Determine the number of possible arrangements of the 10 cars if they are lined up in a row along one side of a parking lot, and a blue car is parked on each end of the row.
16. A six player volleyball team stands together in a straight line for a picture. If two particular players, Joan and Emily, must be together, then how many different arrangements can be made for the picture?

17. At a particular hotel, the following items are available for the continental breakfast.

<table>
<thead>
<tr>
<th>Beverage</th>
<th>Pastry</th>
<th>Fruit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coffee</td>
<td>Muffin</td>
<td>Apple</td>
</tr>
<tr>
<td>Tea</td>
<td>Toast</td>
<td>Orange</td>
</tr>
<tr>
<td>Juice</td>
<td>Doughnut</td>
<td>Grapefruit</td>
</tr>
<tr>
<td></td>
<td></td>
<td>banana</td>
</tr>
</tbody>
</table>

If the continental breakfast consists of 1 beverage, 1 pastry, and 2 different types of fruit, then the number of possible breakfasts that can be ordered is:

A. $\binom{3}{1} \times \binom{3}{1} \times \binom{4}{2}$

B. $\binom{3}{1} \times \binom{3}{1} \times \binom{4}{2}$

C. $\binom{10}{4}$

D. $\binom{10}{4}$

18. In a basketball league, there are 6 teams. In league play, each team must play every other team twice. Determine the number of games that must be scheduled.

19. The vertices of an octagon are marked on a circle. Determine the number of triangles that can be formed using any 3 of the vertices.

20. In a group of 9 people, there are 4 females, and 5 males. Determine the number of 4-member committees consisting of at least 1 female that can be formed.

21. How many different arrangements of the letters TOFIELD can be made using exactly 2 vowels and exactly 2 consonants?
22. Given the diagram to the right, determine the number of pathways, starting from A, and moving to B along the gridlines if a pathway must pass through C and must always move closer to B.

23. For each of the diagrams below, determine the number of pathways starting from A and moving to B along the gridlines given that pathway must always move closer to B.

<table>
<thead>
<tr>
<th>Diagram I</th>
<th>Diagram II</th>
<th>Diagram III</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram I" /></td>
<td><img src="image2.png" alt="Diagram II" /></td>
<td><img src="image3.png" alt="Diagram III" /></td>
</tr>
</tbody>
</table>

24. Determine the coefficient of the term containing $Ax^2y^5$ in the expansion of $(2x + y)^7$.

25. Expand $(2a - 3)^4$ by using the binomial theorem.

26. In the expansion of the binomial $(a + b)^{2k-4}$, there are 15 terms. The value of $k$ is;
   A. 15
   B. 10
   C. 9
   D. 5
27. Four friends are each standing at a different corner of a large maze, as shown to the right. Serena, Bartek, and Anne-Marie remain stationary. Each line segment represents a possible path in the maze. In order for Pierre to travel through the maze to Anne-Marie, he can only move south or east on the paths. If Pierre must avoid the points where Serena or Bartek are standing, how many distinct paths can Pierre take?
A. 248  
B. 250  
C. 252  
D. 254

28. The diagram to the right shows a top view of a set of monkey bars. A child wants to move along the monkey bars from point A to point B. If the child only travels east or south, then the number of different paths she can take from A to B is:
A. 24  
B. 28  
C. 42  
D. 56

29. The value of \((a+b)^0\) defines the first row in Pascal’s triangle. The coefficients of the terms in the expansion of \((a+b)^1\) define the second row, the coefficients of the terms in the expansion of \((a+b)^2\) define the third row, and so on. There is a relationship between the \(n^{th}\) row in Pascal’s triangle and the sum of the values in that row. Which of the following establishes the pattern of numbers for the first 4 rows?
A. 1, 2, 4, 8  
B. 1, 4, 9, 16  
C. 1, 2, 3, 4  
D. 2, 4, 8, 16

30. A student must count the number of paths from point A to point C in the three dimensional diagram shown to the right. If each path must follow along the lines and the path must always get closer to point C, then the number of distinct paths from point A to point C, through point B is:
A. 9  
B. 12  
C. 36  
D. 64
31. Dell has 5 different pairs of jeans, 10 different T-shirts, 3 different pairs of shoes, 2 different watches and 1 jacket. He must choose 1 of each of these items to create an outfit. The number of different outfits he can create is _____.

32. What is the number of distinguishable 5-letter arrangements that are possible using the letters in the word VERTICAL if each arrangement must begin with a vowel?
   A. 360
   B. 840
   C. 2 520
   D. 6 720

33. How many different paths are possible from A to B such that the path always moves closer to B?
   A. 18
   B. 36
   C. 60
   D. 84

34. There are 10 teams entered in a hockey tournament in which each team must play every other team exactly once. How many games will there be altogether in the tournament?
   A. 10!
   B. 2(10!)
   C. \(10P_2\)
   D. \(10C_2\)

35. A school bus has room for 42 children. 28 children are already seated on the bus. If eight additional children need to be seated, how many ways can this be done?
   A. \(14P_8\)
   B. \(14C_8\)
   C. \(14P_6\)
   D. \(14C_8\)

36. How many ways can the shapes shown in the diagram on the right be arranged in the table?
Additional Combinatorics Questions from the Exam Bulletin

1. How many arrangements of all of the letters of the word REASON are there if the arrangement must start with an S?

2. If all of the letters in the word DIPLOMA are used, then how many different arrangements are possible that begin and end with an I, O, or A?
Josh wants to rent a car. He has narrowed his choices to a sedan, a compact, or an economy car. The colours available are black, red, or white. He may also choose between a standard and an automatic transmission.

3. Determine the total number of options Josh has.

Possible solution: 18

<table>
<thead>
<tr>
<th>Category</th>
<th>Colour</th>
<th>Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sedan</td>
<td>Black</td>
<td>Standard</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Automatic</td>
</tr>
<tr>
<td></td>
<td>Red</td>
<td>Standard</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Automatic</td>
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<tr>
<td></td>
<td>White</td>
<td>Standard</td>
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<td>Automatic</td>
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<tr>
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<td>Economy</td>
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<td>Standard</td>
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<td></td>
<td>Automatic</td>
</tr>
</tbody>
</table>
A volleyball team made up of 6 players stands in a line facing the camera for a picture.

4. If Joan and Emily must be together, then how many different arrangements are possible for the picture?

5. Determine the number of different arrangements using all the letters of the word ACCESSSES that
   a) begin with exactly two S’s.
   b) begin with at least two S’s.
   c) Explain why the answers in questions (a) and (b) are different.

Use the following information to answer the next question.

At a car dealership, the manager wants to line up 10 cars of the same model in the parking lot. There are 3 red cars, 2 blue cars, and 5 green cars.

6. If all 10 cars are lined up in a row facing forward, determine the number of possible car arrangements if the blue cars cannot be together

Use the following information to answer the next question.

If 14 different types of fruit are available, how many different fruit salads could be made using exactly 5 types of fruit?

Student 1  Kevin used \(\binom{14}{5}\) to solve the problem.

Student 2  Ron suggested using \(14P_5\).

Student 3  Michelle solved the problem using \(14C_5\).

Student 4  Jackie thought that \(5!\) would give the correct answer.

Student 5  Stan decided to use \(\binom{14}{5}\).

7. The correct solution would be obtained by student number _____ and student number _____.

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Use the following information to answer the next question.

At a meeting, every person shakes hands with every other person exactly once.

8. If there are 36 handshakes in total, how many people were at the meeting?

9. How many different 4-letter arrangements are possible using any 2 letters from the word SMILE and any 2 letters from the word FROG?

10. Find the value of \(a\) if the expansion of \((2x + 3)^{3a - 5}\) has 18 terms.

Use the following information to answer the next question.

A student made the following statements regarding the expansion of \((a + b)^4\), written in descending powers of \(a\).

Statement 1 The total number of terms is 5.
Statement 2 The middle term is \(6a^2b^2\).
Statement 3 The sum of the leading coefficients of all the terms is 14.
Statement 4 For the term \(4a^3b^m\), the value of \(m\) is 1.
Statement 5 The leading coefficient of the first term is \(4C_1\).

11. The three statements that are true are numbered _____, _____, and _____.

12. In the expansion of \((3a - b^2)^{10}\), what is the coefficient of the term containing \(a^4b^{12}\)?
Use the following information to answer the next question.

In the expansion of the binomial \( (2a + \frac{1}{a})^8 \), the constant term can be expressed in the form \( \binom{n}{k} x^{n-k} y^k \). The list below shows possible values for \( n, k, x, \) and \( y \).

<table>
<thead>
<tr>
<th>Reference Number</th>
<th>Possible Values of ( n, k, x, ) and ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>2a</td>
</tr>
<tr>
<td>7</td>
<td>( a )</td>
</tr>
<tr>
<td>8</td>
<td>( a^{-1} )</td>
</tr>
<tr>
<td>9</td>
<td>( -a )</td>
</tr>
</tbody>
</table>

13. Using the reference numbers, the correct values of \( n, k, x, \) and \( y \) that must be used to find the constant term are numbered, respectively, _____, _____, _____, and _____.

14. Given that a term in the expansion of \( (ax - y)^6 \) is \(-252xy^5\), determine the numerical value of \( a \).
Polynomial, Rational and Composition Functions

1. Given a polynomial $P(x)$, what condition must be true for $x - 1$ to be a factor of $P(x)$?

A. $P(-1) = 0$
B. $P(1) = 0$
C. $P(x) = 1$
D. $P(x) = -1$

2. If $(x + 2)$ is a factor of $3x^3 + kx^2 - 31x - 54$, find $k$.

A. $-8$
B. $-4$
C. $\frac{102}{2}$
D. $4$

3. Which of the following statements is true for the relationship between $x + 2$ and $P(x) = 2x^3 - 5x + 6$?

A. Since $P(2) \neq 0$, $x + 2$ is a \textit{not} a factor of $P(x)$.
B. Since $P(2) = 0$, $x + 2$ is a factor of $P(x)$.
C. Since $P(-2) \neq 0$, $x + 2$ is a \textit{not} a factor of $P(x)$.
D. Since $P(-2) = 0$, $x + 2$ is a factor of $P(x)$. 
4. If \( P(x) = x^3 - 2x^2 + 9x - 2 \) which of the following statements are true?

   I. \( x - 3 \) is a factor of \( P(x) \)
   II. \( x = 3 \) is a root of \( P(x) = 0 \)
   III. \( P(3) = 34 \)
   IV. \( P(-3) = 34 \)

   A. III only
   B. I and II only
   C. I and III only
   D. IV only

5. How many of the following statements are true?

   - \( x - 1 \) is a factor of \( 2x^{10} - x^7 - 1 \).
   - The remainder for \( (2x^{10} - x^7 - 1) \div (x - 1) \) is 0.
   - If \( P(x) = 2x^{10} - x^7 - 1 \) then \( P(1) = 0 \).

   A. 1
   B. 3
   C. 0
   D. cannot be determined

6. Which of the following polynomial function written in factored form best represents the graph and table given in the figure?

   A. \( P(x) = -\frac{1}{2}(x + 3)(x - 1)^2 \)
   B. \( P(x) = \frac{1}{2}(x + 3)(x - 1)^2 \)
   C. \( P(x) = \frac{1}{2}(x - 1)^3 \)
   D. \( P(x) = -\frac{1}{2}(x - 1)^3 \)
7. When a polynomial $P(x)$ is divided by $2x + 1$ the quotient is $x^2 - x + 2$ and the remainder is 5. What is $P(x)$?

A. $P(x) = 2x^3 - x^2 + 3x + 7$
B. $P(x) = 2x^3 - x^2 + 3x - 7$
C. $P(x) = 2x^3 - x^2 + 7x - 7$
D. $P(x) = 2x^3 - 3x^2 + 7x - 7$

8. If $P(x) = 2x^4 - 2x^3 + Ax + B$, find $A$ and $B$ such that $P(-2) = 36$, and $P(2) = 24$.

A. $A = -3, \ B = 6$
B. $A = -3, \ B = -6$
C. $A = 1, \ B = 2$
D. $A = -3, \ B = -2$

9. Write the division statement for $(x^5 - 3x^3 + 2x - 8) \div (x - 4)$.

A. $(x^4 + 4x^3 + 13x^2 + 52x + 210)(x - 4) + 832$
B. $(x^4 + x^2 + 6)(x + 4) - 16$
C. $(x^4 + x^2 + 6)(x - 4) + 32$
D. $(x^4 + x^2 + 6)(x - 4) + 16$

10. Determine the quotient when $x^3 - 2x^2 - 9$ is divided by $x - 3$.

A. $x^2 + 5x + 15$
B. $x^2 + x - 6$
C. $x^2 + x + 3$
D. $x^2 - x - 6$
11. Find the quotient and remainder of \((x^3 + 8x^2 + 19x + 13) \div (x + 3)\).

   A. \((x^2 + 11x + 52) \text{ R } 13\)
   B. \((x^2 + 5x + 4) \text{ R } -1\)
   C. \((x^2 + 11x + 52) \text{ R } 169\)
   D. \((x^2 + 5x + 4) \text{ R } 1\)

12. Use synthetic division to find the remainder when \(2x^4 + 3x^3 - x^2 - 2\) is divided by \(x + 2\).

   A. 2
   B. -4
   C. 4
   D. -2

13. When \(x^2 + 3x + b\) is divided by \(x + a\), the quotient is \(x - 2\) and the remainder is 7. Find \(a\) and \(b\).

   A. \(a = -5, \; b = 3\)
   B. \(a = -3, \; b = -4\)
   C. \(a = 3, \; b = 4\)
   D. \(a = 5, \; b = -3\)

14. What is the maximum number of real roots that the following equation could have? (Do not solve.)

   \[x^3 + 2x^2 - 7x + 5 = 0\]

   A. 1
   B. 2
   C. 3
   D. none
15. For a certain polynomial function \( P \), the following values are given: \( P(-2) = -7 \), \( P(-1) = -2 \), \( P(0) = -1 \), \( P(1) = 3 \) and \( P(2) = 12 \). This function has at least one real zero between:

A. -2 and -1  
B. -1 and 0  
C. 0 and 1  
D. 2 and 3

16. Find the real roots of \( x^3 + 2x^2 - x - 2 = 0 \) using the Factor Theorem.

A. -2, \( \pm 1 \)  
B. 1, \( \pm 2 \)  
C. 2, \( \pm 1 \)  
D. \( \pm 1, \pm 2, \pm 4 \)

17. \( P(x) = 0 \) is a polynomial equation whose roots are \(-1\), 2, and \(-2\). If \( P(0) = 12 \), find the function \( P(x) \).

A. \(-3x^3 - 3x^2 - 12x + 12\)  
B. \(-3x^3 - 3x^2 + 12x + 12\)  
C. \(-3x^3 + 3x^2 - 12x + 12\)  
D. \(3x^3 - 3x^2 + 12x + 12\)
18. Which of the following is the graph of \( y = a(x + 1)(x - 2)(x - 2) \), where \( a > 0 \)?

A. 

B. 

C. 

D. 

19. A cubic equation has roots of \(-3, -1,\) and \(2\). If its \(y\)-intercept is 24, then what is its (fully expanded) equation?

A. \( y = -\frac{5}{2}x^3 + 5x^2 + 9\frac{5}{2}x - 25 \)

B. \( y = -\frac{5}{2}x^3 - 5x^2 + 9\frac{5}{2}x - 25 \)

C. \( y = -4x^3 - 8x^2 + 20x + 24 \)

D. \( y = -4x^3 + 8x^2 + 20x - 24 \)
20. Which of the following graphs best illustrates the graph of a fifth degree polynomial function whose leading coefficient is positive?

A.  

B.  

C.  

D.  

21. Find the horizontal asymptote of \( y = \frac{7}{x-4} \).

A.  \( x = 4 \)
B.  \( y = 0 \)
C.  \( x = 0 \)
D.  \( y = 4 \)
22. Find the vertical asymptote(s) of \( y = \frac{x^2}{x^2 - 1} \).

A. \( x = \pm 1 \)
B. \( y = x \)
C. \( y = 1 \)
D. \( x = 0 \)

23. Let \( f(x) = \frac{1}{(x - 4)^3} \). What are the asymptotes of \( f(x) \)?

A. \( y = 0 \) only
B. \( x = 4 \) only
C. \( x = 1 \) and \( y = 0 \)
D. \( x = 4 \) and \( y = 0 \)

24. The graph of \( y = \frac{x^2 + 3x + 2}{x^2 + 2x - 3} \) has which of the following asymptotes?

A. \( x = 1, y = -3, \) and \( y = 1 \)
B. \( y = 1, x = 3, \) and \( x = 1 \)
C. \( y = -1, x = -3, \) and \( x = 1 \)
D. \( y = 1, x = -3, \) and \( x = 1 \)

25. Let \( f(x) = \frac{x^3 + 5x - 4}{x(x - 1)} \). What are the asymptotes of \( f(x) \)?

A. \( x = 1 \) only
B. \( x = 0, x = 1, \) and \( y = x \)
C. \( x = 0, x = 1, \) and \( y = x - 1 \)
D. \( x = 0, x = 1, \) and \( y = x + 1 \)
26. Given a function defined by \( f(x) = \frac{3x - 12}{x^2 - 6x + 8} \), for what value(s) of \( x \) is the function discontinuous?

A. 4 only
B. 2
C. 2, 4
D. −4, −2

27. Let \( f(x) = 7x + 2 \) and \( g(x) = x^2 - 9 \). Find \( (f \times g)(x) \).

A. \( 7x^3 + 2x^2 - 63x - 18 \)
B. \( 7x^3 - 18 \)
C. \( 2x^2 + 7x - 7 \)
D. \( x^2 + 7x - 7 \)

28. Let \( f(x) = 6x - 12 \) and \( g(x) = x^2 - 4 \). Find \( \left( \frac{f}{g} \right)(x) \).

A. \( \frac{6(x - 2)}{x + 2} \)
B. \( \frac{6}{(x - 2)(x + 2)} \)
C. \( \frac{6}{x + 2} \)
D. \( \frac{6}{x + 3} \)
29. Suppose \( f(x) = 7x^2 - 3 \) and \( g(x) = 9 - 2x \). Find \((g \circ f)(x)\).

A. \( 60 - 14x \)

B. \( 15 - 14x^2 \)

C. \( 7x^2 - 2x + 6 \)

D. \( 63x^2 + 6x - 27 \)

30. Suppose \( f(x) = x + 7 \) and \( g(x) = \frac{x - 3}{8} \). Find \((g \circ f)(x)\).

A. \( x + 1 \)

B. \( \frac{x - 53}{8} \)

C. \( \frac{x + 4}{8} \)

D. \( x - 53 \)

31. Suppose \( f(x) = 1 - x^2 \) and \( g(x) = \frac{1}{\sqrt{x}} \). Find \( f(g(x)) \).

A. \( \frac{1 - x^2}{\sqrt{x}} \)

B. \( \frac{1}{\sqrt{1 - x^2}} \)

C. \( 1 - \frac{1}{x} \)

D. \( \frac{1 - x}{\sqrt{x}} \)
32. If \( g(f(x)) = 9 - 6x, f(x) = 3x - 2, \) and \( g(x) = ax + b, \) then \( g(x) = \) _____

A. \( 21 - 12x \)
B. \( 10 - 2x \)
C. \( 10 - x \)
D. \( 5 - 2x \)

33. The roots of \( y = P(x) \) are as shown. What are the roots of \( y = 2P(x - 5)? \)

A. \( -14, -12, -8 \)
B. \( -4, -2, 2 \)
C. \( 3, 4, 6 \)
D. \( 6, 8, 12 \)

Given \( f(x) = 2x + 5, \) \( g(x) = x^2 - 10 \) and \( h(x) = 3x - 8, \) find the following.

34. \( f(h(g(-7))) \)
35. \( h(f(g(-6))) \)
36. \( h(f(x)) \)

37. Which of the following is the correct domain and \( y \)-intercept of the function \( y = \frac{4}{4x^2 - b^2}? \)

A. \( x \neq b^2, \) \( y = 4 \)
B. \( x \neq \frac{\pm b}{2}, \) \( y = 4 \)
C. \( x \neq b^2, \) \( y = \frac{-4}{b^2} \)
D. \( x \neq \frac{\pm b}{2}, \) \( y = \frac{-4}{b^2} \)

38. The function \( y = \frac{5x^2 - 12x + 4}{x-a} \) is undefined at the point \((a, b)\). The value of \( b \) is _____.
Additional Polynomial, Rational and Composition Functions Questions from the Exam Bulletin

39. Express the following polynomials in factored form.
   
a) \( P(x) = x^3 - x^2 - 8x + 12 \)

b) \( P(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9 \)

40. Which of the following is a factor of \( f(x) = 4x^4 - x^3 - 8x^2 - 40 \)?
   
   A. \((x + 2)\)  
   B. \((x - 4)\)  
   C. \((x - 5)\)  
   D. \((x + 8)\)

41. If \( x + 2 \) is a factor of \( f(x) = x^2 + 3x^2 - 4k + 4 \), determine the value of \( k \).

42. If \( P(x) \) is a polynomial function where \( P\left(-\frac{2}{3}\right) = 0 \) and \( P(0) = 12 \), then \( i \) is a factor of \( P(x) \), and \( ii \) is a constant term in the equation of \( P(x) \).

The statement above is completed by the information in row

<table>
<thead>
<tr>
<th>Row</th>
<th>i</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>((3x - 2))</td>
<td>12</td>
</tr>
<tr>
<td>B.</td>
<td>((3x + 2))</td>
<td>12</td>
</tr>
<tr>
<td>C.</td>
<td>((3x - 2))</td>
<td>-12</td>
</tr>
<tr>
<td>D.</td>
<td>((3x + 2))</td>
<td>-12</td>
</tr>
</tbody>
</table>

Use the following information to answer the next question.

A list of five functions is given below.

<table>
<thead>
<tr>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( y = x^4 + 10x^3 - 2x + 5 )</td>
</tr>
<tr>
<td>2 ( y = 3x^3 - 2x^2 + x^{-1} - 4 )</td>
</tr>
<tr>
<td>3 ( y = \sqrt[5]{x} + 3 )</td>
</tr>
<tr>
<td>4 ( y = 4x^3 + 2x^2 + \frac{1}{x} )</td>
</tr>
<tr>
<td>5 ( y = -2x^3 + 7x^2 - 3x^3 + 2^4 - 7 )</td>
</tr>
</tbody>
</table>

43. Which of the functions above represents a polynomial function? Explain why or why not.
44. Sketch the graph of a fifth degree polynomial function with one real zero of multiplicity 3 and with a negative leading coefficient.

45. Given the function \( y = \frac{1}{4}(x - 2)(2x + 5)(x + 4)^2 \),

a) Accurately sketch the graph, and label any key points.

b) State the domain and range.

c) Determine the zeros of the function.

*Use the following information to answer the next question.*

The graph of the polynomial function \( y = f(x) \) is shown below.

![Graph of polynomial function](image)

46. What is the minimum possible degree for the polynomial function above? Determine an equation of the function in factored form.
Use the following information to answer the next question.

The graphs of four polynomial functions are shown below.

Graph 1

Graph 2

Graph 3

Graph 4

47. Match three of the graphs numbered above with a statement below that best describes the function.

The graph that has a positive leading coefficient is graph number _________.

The graph of a function that has two different zeros, each with multiplicity 2, is graph number _________.

The graph that could be a degree 4 function is graph number _________.

Use the following information to answer the next question.

A box with no lid is made by cutting four squares of side length $x$ from each corner of a 10 cm by 20 cm rectangular sheet of metal.

48. Using the information above, follow the directions below.

a) Find an expression that represents the volume of the box.
b) Sketch the graph of the function and state the restriction.

c) Find the value of $x$, to the nearest hundredth of a centimetre, that gives the maximum volume.

d) What is the maximum volume of the box, to the nearest cubic centimetre?

49. Determine the roots of the equation $2x^3 - 3x^2 - 10x + 3 = 0$. Leave answers as exact values.

53. Sketch the graph of the following functions and determine the following characteristics for each function below: domain, $x$- and $y$-intercepts, equation of vertical asymptotes.

   a) $y = \frac{3x}{x^2 + 2x - 8}$

   b) $y = \frac{x + 3}{x^2 - 9}$

54. For the graph of $y = \frac{3x + 7}{2x + 5}$, determine the equation of the horizontal asymptote and the range.

55. Determine the coordinates of the point of discontinuity on the graph of $f(x) = \frac{2x^2 - 15x + 7}{x - 7}$. 

66
Use the following information to answer the next question.

The graph of the function below can be expressed in the form \( y = \frac{ax}{x^2 + bx + c} \).

56. Determine the values of \( a \), \( b \), and \( c \).

1. Given the functions \( f(x) = 7 - x \) and \( g(x) = 2x + 1 \), sketch the graph of \( h(x) \) for each question below and state the domain and range.

   a) \( h(x) = f(x) - g(x) \)

   b) \( h(x) = f(x) g(x) \)

   c) \( h(x) = \left( \frac{g}{f} \right)(x) \)

   d) \( h(x) = g(f(x)) \)

2. Given \( f(x) = \sqrt{x - 1} \), \( g(x) = x^2 + 3 \), and \( h(x) = 2x - 5 \), determine \( k(x) = (h \circ g \circ f)(x) \), including restrictions on the domain.
Use the following information to answer the next question.

The graphs of the functions \( y = f(x) \) and \( y = g(x) \) are shown below.

3. Sketch the graph of
   a) \( h(x) = f(x) + g(x) \)
   b) \( h(x) = \frac{f(x)}{g(x)} \)

4. Given \( f(x) = 7 \log_2 x \) and \( g(x) = |5x - 6| \), determine the value of \( f'(g(-2)) \).

Use the following information to answer the next question.

Alex is given the following list of functions, where \( b > 1 \). She is asked to determine a new function, \( h(x) \), which is the quotient of two different functions where \( h(x) \) requires no restriction.

| Function 1  | \( y = b^x \) |
| Function 2  | \( y = \log_b x \) |
| Function 3  | \( y = \sqrt{x} \) |
| Function 4  | \( y = x^2 \) |

5. If \( h(x) = \frac{a}{b} \), then \( a \) is the function numbered _____ and \( b \) is the function numbered _____.

6. Given \( f(x) = x^2 - 5 \), \( g(x) = x - 2 \), and \( h(x) = \frac{2x^2 - 6x + 5}{x - 2} \), determine \( j(x) = \frac{f(x)}{g(x)} + h(x) \). State the domain and range of \( j(x) \).