Grade 12 pre-calculus mathematics achievement test.
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Available in alternate formats upon request.
Instructions

Multiple-Choice Questions

- There are 9 questions each worth one mark.
- Calculators are not allowed for this part of the test.
- You may use the spaces beside each question for rough work.
- Provide only one answer per question.
- There is no penalty for guessing.
- Record your answers on the sheet provided.

Short and Long Answer Questions

- There are 25 questions worth a total of 49 marks.
- Calculators are not allowed for this part of the test.
- For full marks, your answer must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear, and well organized.
- Write each solution in the space provided.
No marks will be awarded for work done on this page.
Question 16 1 mark

If \((2, 3)\) is a point on the graph of \(y = f(x)\), what point must be on the graph of \(y = 3f\left(\frac{1}{4}x\right)\)?

a) \(\left(\frac{1}{2}, 1\right)\)  b) \(\left(\frac{1}{2}, 9\right)\)  c) \((8, 1)\)  d) \((8, 9)\)

Question 17 1 mark

Consider the arc drawn on each circle. Which arc measure is closest to 3 radians?

a) \[
\begin{array}{c}
\text{a)} \\
\end{array}
\]

b) \[
\begin{array}{c}
\text{b)} \\
\end{array}
\]

c) \[
\begin{array}{c}
\text{c)} \\
\end{array}
\]

d) \[
\begin{array}{c}
\text{d)} \\
\end{array}
\]

Question 18 1 mark

If \(\log_2 x = 4\), then \(\log_2 (2x)\) is equal to:

a) 5  b) 8  c) 16  d) 32
Question 19  

Simplify the following expression:
\[ \cos^2 x \left( 1 + \cot^2 x \right) \]

a) \( \sin^2 x \)  

b) \( \cos^2 x \)  

c) \( \cot^2 x \)  

d) \( \sec^2 x \)  

Question 20  

Identify the graph of the function \( y = \frac{x}{x} \).  

a)  

b)  

c)  

d)
Question 21

How many terms are in the expansion of \((3y^2 - 4z)^7\)?

a) 2  

b) 6  

c) 7  

d) 8

Question 22

Determine one possible restriction for the domain of \(y = (x + 3)^2 - 4\) so that its inverse is a function.

a) \(x \leq -3\)  

b) \(x \leq 0\)  

c) \(x \leq 3\)  

d) \(x \leq 4\)

Question 23

Find the total possible number of arrangements for 7 adults and 3 children seated in a row if the 3 children must sit together.

a) \(10!\)  

b) \(8!3!\)  

c) \(7!3!\)  

d) \(7!\)

Question 24

Identify the value of the \(x\)-intercept of the function \(y = \ln(x - 2)\).

a) \(-1\)  

b) \(0\)  

c) \(2\)  

d) \(3\)
Question 25 1 mark

Given \( \log_b a = 3 \), give one example of possible values for \( a \) and \( b \) that make this equation true.

Question 26 1 mark

The range of the graph of \( y = f(x) \) is \( [-3, 2] \).

Explain why there is no effect on the range of the graph that is a result of the transformation \( y = f(-x) \).
Sketch the graph of \( y = (x + 1)(x - 2)^2(x + 5) \).

Identify the \( x \)-intercepts and \( y \)-intercept.

\( x \)-intercepts: \\

\( y \)-intercept:
Question 28 2 marks

The graph of the function \( y = \sin x \) has been transformed to create a new graph.

The range of this new graph is \([-4, 4]\) and the zeros are \( x = k \frac{\pi}{2} \), where \( k \) is an integer.

Write the equation that corresponds to this new graph.

Question 29 1 mark

Given the functions \( f(x) = x^2 - 1 \) and \( g(x) = x + 1 \), state the domain of \( \frac{g(x)}{f(x)} \).
Question 30

a) Sketch the graph of \( y = 3^x \).

b) Explain how the graph of \( y = 3^x \) can be used to sketch the graph of \( y = \log_3 x \).
Question 31 5 marks

A box in the shape of a rectangular prism has side lengths $x$, $x + 2$, and $x + 10$.
Write a function, $V(x)$, to express the volume of the box in terms of $x$.
Find all possible values of $x$, given that the volume of the box is 96 $\text{cm}^3$.
State the dimensions of the box.
**Question 32**

Given the graph of \( f(x) \) below, sketch the graph of \( y = -f(x) \).

![Graph of \( f(x) \) and \( y = -f(x) \)]

**Question 33**

Determine the coordinates of a point \((x, y)\) on the unit circle if you are given \( \theta = 30^\circ \) where \( \theta \) is in standard position.
Given the following sinusoidal equation:

\[ P(t) = 3000 \sin \left( \frac{\pi}{10} (t - 2010) \right) + 10000 \]

Determine the maximum value of \( P(t) \) and a value of \( t \) at which this maximum occurs.

Max value of \( P(t) \): __________________

Value of \( t \): __________________
Question 35  

Sketch the graph of $y = \sqrt{2x - 2}$. 
Question 36 2 marks

Given \( f(x) = 2x - 6 \), write the equation of \( f^{-1}(x) \).

Question 37 1 mark

Frank tried to expand a logarithmic expression using the laws of logarithms. He made one error.

Frank's solution: \( \log_a \left( \frac{x + 2}{zw} \right) = \log_a x + \log_a 2 - \log_a z - \log_a w \)

Write the correct solution.
Question 38

Determine all non-permissible values of \( \theta \) over the interval \([0, 2\pi]\).

\[
\frac{\sin \theta}{1 + \cos \theta} + \csc \theta + \cot \theta
\]

Explain your reasoning.
Given the following graphs:

![Graph of f(x)](image)

![Graph of g(x)](image)

a) Determine the value of \(f \cdot g(0)\).

b) Determine the value of \(g(f(4))\).

c) Determine a value for \(k\) where \(f(k) = 1\).
Question 40

Given that \( h(x) = 2x^2 + 5x - 3 \) and that \( h(x) = f(x) \cdot g(x) \), determine \( f(x) \) and \( g(x) \).

Question 41

The graph of \( y = 2\cos \theta + 1 \) below can be used to solve the equation \( \cos \theta = -\frac{1}{2} \) over the interval \([-2\pi, 2\pi]\). Indicate on the graph where to find the solutions to the equation \( \cos \theta = -\frac{1}{2} \).
Question 42

The function \( f(x) \) is transformed.

A new function, \( y = \frac{1}{f(x)} \), is created that does not have any vertical asymptotes.

What can you conclude about the original function \( f(x) \)?

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Question 43

Draw the angle \(-\frac{7\pi}{8}\) in standard position.

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Question 44

Determine the exact value of:

\[ 4 \cos \left( \frac{11\pi}{12} \right) \]
Question 45  

3 marks

Sketch the graph of \( f(x) = \frac{x - 4}{x^2 - 3x - 4} \).
Question 46

Estimate the value of \( \log_5 35 \).
Justify your answer.

Question 47

If \( p(x) = x^5 - 12x + 1 \), determine the remainder when \( p(x) \) is divided by \( (x + 2) \).
Question 48 1 mark

Describe the effects on the graph of $y = f(x)$ when asked for the graph of $y = f(x - 3) + 5$.

Question 49 3 marks

Find the exact value of the following expression:

$$\sin\left(\frac{11\pi}{3}\right) \cdot \sec\left(\frac{4\pi}{3}\right) \cdot \tan\left(-\frac{5\pi}{6}\right)$$
No marks will be awarded for work done on this page.
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